Radiative and Non-Leptonic Decays of Bottom Baryons in the Quark Model

by

Mohamed-Rabigh Khodja

A Thesis Presented to the

FACULTY OF THE COLLEGE OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

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This thesis, written by Mohamed-Rabigh Khodja under the direction of his Thesis Advisor and approved by his Thesis Committee, has been presented to and accepted by the Dean of the College of Graduate Studies, in partial fulfillment of the requirements for the degree of **MASTER OF SCIENCE IN PHYSICS.**

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كلية الدراسات العليا
COLLEGE OF GRADUATE STUDIES

إلى والـدَيّ الكريمـين،

إلى أُمّة مـرزاقـة،

إلى أخـواتـي،

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ABSTRACT

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This study is divided into two parts. In the first part, the two-body radiative decays of the b-flavored baryons are analyzed in the framework of the nonrelativistic quark model and perturbation theory. With the obtained decay widths, we give an estimate for the mean lifetimes of the states $\Xi_b^{0\prime}$, $\Xi_b^{-\prime}$, and Ω_b^{*-}. As for the second part, the two-body nonleptonic decays of bottom baryons containing a single b-quark are investigated in the factorization scheme, treating the color-matching parameter ξ as a free parameter. The nonrelativistic quark model is used to calculate the weak current baryonic form factors at $s = m_X^2$, where X is either a pseudoscalar, a vector or an axial vector meson, thereby taking into account the recoil correction. The obtained decay rates are not very sensitive to ξ, if $0 \leqslant \xi \leqslant 0.5$ as suggested by two-body B meson decays.

MASTER OF SCIENCE DEGREE

KING FAHD UNIVERSITY OF PETROLEUM AND MINERALS

DHAHRAN, SAUDI ARABIA

ABSTRACT

خُلاصة الرّسالة

الاسم: محمد الرَّبيع بن عبد الحميد بن محمد الرَّبيع خُوجَه

العنوان: دراسةٌ للتفكّكات الإشعاعية وغير اللّبتونيّة لباريُونات تحوي قُورْكًا واحدًا (b) باستعمال أُنموذج القَوارك

التخصص: فيزياء

التاريخ: صفر ١٤١٩هـ

تنقسم هذه الرسالة إلى قسمين. تمّ في القسم الأول منها دراسة التفككات الإشعاعية للباريونات التي تحوي قُورْكًا واحدًا (b) باستعمال أنموذج القوارك غير النسبويّ و نظرية الاضطرابات الكَمّيَّة. فتمّ حساب معدّلات التفكك الإشعاعي لهذه الباريونات، و من ثمّ متوسط أعمار الجسيمات (Ω_b^{*-})، $(\Xi_b^{\prime 0})$ و $(\Xi_b^{\prime -})$. أما فيما يتعلق بالقسم الثاني، فلقد قمنا بمعالجة التفككات غير اللّبتونيَّة للباريونات التي تحوي قُورْكًا واحدًا (b)، بافتراض قابلية عناصر مصفوفات التفكك للتحليل إلى جُداء و اعتبار الحدّ (ξ) وسيطًا طَلْقًا. كما تمّ استخدام أنموذج القوارك غير النسبويّ لحساب عوامل الهيئة الباريونيَّة عند ($s = m^2_X$) حيث يكون (X) إما ميسونا شعاعيا أو ميسونا شبه شعاعي أو ميسونا شبه سلمي. آخذين بذلك في الاعتبار تصحيح التراجع. فنتج لدينا أن معدلات التفكك المحسوبة لا تتأثر كثيرا بالوسيط (ξ) إذا كان يقع في المجال $(0 \le \xi \le 0.5)$ كما تشير إليه نتائج تفكك الميسونات (B).

درجة الماجستير في العلوم

جامعة الملك فهد للبترول و المعادن

الظهران، المملكة العربية السعودية

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Chapter 1

Introduction

Heavy-quark physics started in 1974 with the observation of the J/Ψ meson, a narrow resonance at a mass of 3.097 GeV. The J/Ψ was quickly identified as a bound state of a charm and anti-charm quark, a previously unobserved quark flavor with a mass around 1.5 GeV.

Charm quark (c-quark) was not only the first heavy-flavor quark to be discovered, it was also the first quark to be theoretically predicted before being discovered. In 1970, Glashow, Iliopoulos, and Maiani put forward the so called GIM scheme [1] as an explanation for the absence of strangeness-changing neutral currents, whose existence was a nasty prediction of the Glashow-Salam-Weinberg model of electroweak unification; the key point of the GIM mechanism is the existence of a new type of quark[1] .

[1] An example of flavor-changing neutral currents, which have never been observed, is provided by the decay of a long-lived neutral kaon $K_L^0 \rightarrow \mu^+\mu^-$. Roughly speaking, by assuming the existence of another quark the GIM mechanism arranges that the unwanted flavor-changing neutral current

In 1977, the second heavy flavor, the *bottom* (b) or *beauty* quark (with contrast to charm) with a mass of approximately 5 GeV and an electric charge of $-1/3$, was reported at Fermilab in the form of a bound state, the Υ (upsilon) meson [2]. The recent observation of the top quark (t-quark) [3]-[4] completes the three quark families of the *standard model* (SM):

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$

According to their masses, the six quarks group into two categories: heavy flavors and light flavors. Heavy flavors c, b, and t have masses larger than the QCD scale, $\Lambda_{QCD} \simeq 250\ MeV$; while the light flavors, i.e., the u, d, and s quarks, have masses smaller than Λ_{QCD} (cf. Table 1.1).

1.1 Heavy Flavor Decays and the SM

The decays of heavy quarks serve as tests for the standard model. For instance the electromagnetic processes are a key tool in the study of hadronic structure. They are used to establish the states, their spin, parity and energy. Of paramount importance is their utilization to extract more information on the wave functions of the states. Such knowledge is of considerable help in validating any theory of hadron structure. The theory which we consider in this study is the *constituent quark model* (CQM) [5]-[7]. This theory has proven quite useful in explaining the observed baryon and meson spectra as well as their static properties (e.g., magnetic moments) both in

diagrams are cancelled out by the corresponding diagrams involving the new quark.

light and heavy sectors [8]-[15]. On the other hand, the weak decays of heavy quarks can be used to determine the parameters of the *SM*, including the mixing angles of the Cabibbo-Kobayashi-Maskawa (*CKM*) matrix [5]-[6]. In addition, weak decays provide us with important insight into one of the least understood phenomena in physics, i.e., charge-parity (*CP*) violation. Although readily accommodated in the *SM* by a complex phase in the *CKM* matrix, the complications introduced by strong interaction effects make it nearly impossible to ascertain whether the complex *CKM* phase is the unique source of the observed phenomenon.

1.2 Weak Decays of Bottom Quarks

The b-quark decays through the weak charged current into a light quark with a charge of $+2/3$, i.e., c or u quark. The coupling is proportional to the element V_{bq} of the *CKM* mixing matrix, where q denotes the final quark, either c or u. Weak decays can, in general, be subdivided into three modes according to the final state particles produced. These are *leptonic*, *semileptonic*, and *nonleptonic* (or *hadronic*). In leptonic decays, the final state contains leptons only. Semileptonic decays produce final states with both leptons and hadrons. Nonleptonic decays contain no leptons in the final state. Nonleptonic decays are far more complex than semileptonic and purely leptonic processes, because of the eminent presence of the strong interaction among the hadrons produced. They involve an intricate interplay of quark rearrangements due to soft and hard gluon exchanges; the hadrons in the final state can rescatter into one another[2] . These processes are called *final state interactions* (*FSI*).

The lowest order nonleptonic decay diagrams for bottom baryons are shown in Figures 1.1-1.4 [7]. The *spectator diagrams*, in which the light quarks do not take part in the weak interaction, are shown in Figures 1.1-1.3, while Figure 1.4 shows the *W-exchange* mechanism. The contribution of diagrams other than the external spectator diagram is expected to be significant for decays of baryons with heavy quarks.

1.3 Effective Theories

In the investigation of physical phenomena one is frequently lead to make approximations. It is usually counterproductive to consider a given problem in the context of an "exact theory". A level of description that is most adequate to the problem at hand is usually the only way out of the perplexing complexity of real phenomena. It is in this context that effective theories are used. A good example is *Fermi's theory* of weak interactions. In this theory the weak decays of hadrons are treated as pointlike 4-fermion interactions. Only at high enough energies does one need to resort to a level of description for which the W and Z vector bosons of electroweak unification are required. Another example of interest to us is the *heavy-quark effective theory* (*HQET*) of Isgur and Wise [8]-[9]). *HQET* starts with model-independent relations that are valid in the limit $1/m_Q \to 0$ and includes the leading symmetry-breaking

[2] For example, a D^0 meson (a bound $c\overline{u}$ state) can decay directly into $\overline{K^0}\pi^0$ or rescatter via the intermediate state $K^-\pi^+$, since $K^-\pi^+ \longrightarrow \overline{K^0}\pi^0$ is an allowed strong interaction.

corrections in a systematic expansion in $1/m_Q$. Indeed, in the limit of infinite quark masses, the dynamics of a heavy quark is independent of its mass and spin. Consequently, in the sector of hadrons containing one heavy quark a new flavor and spin symmetry appear. The spin and mass of the heavy quark are no longer relevant to the description of the hadron. The heavy quark acts merely as a source of color charge. This is traditionally likened to the irrelevance of the mass and spin of the nucleus for the description of a hydrogenlike atom. In the last few years, considerable progress has been achieved in the understanding of the weak decays of heavy hadrons due to the development of *HQET* [21]-[23].

Another important concept, which we encounter in this study, is that of *factorization* . Factorization is the hypothesis that the probability amplitude of a decay can be expressed as the product of two single current matrix elements. In the case of hadronic decays, this hypothesis is applied by analogy to semileptonic decays where the amplitude can be decomposed into a leptonic and a hadronic current. Physically, factorization amounts to neglecting the influence of the final states of a process on each other, i.e., to neglecting *FSI*: in a two body decay, for instance, one might think of this, intuitively, as one of the particles traveling fast enough to leave the interaction region before exchanging any information with the other particle. Factorization has been shown to be valid in the limit $\frac{1}{N_c} \to 0$, N_c being the number of colors, and $\frac{1}{N_c}$ corrections to this limit have been considered [10]. Moreover, it has been succesfully applied to heavy-to-heavy nonleptonic B meson decays [25]-[33].

q	u	d	s	c	b
$m(GeV)$	0.338	0.322	0.510	1.6	4.9
Charge	+2/3	−1/3	−1/3	+2/3	−1/3
Bottomness	0	0	0	0	−1
Charm	0	0	0	−1	0
Strangeness	0	0	−1	0	0

Table 1.1: Masses, electric charges and flavor quantum numbers of constituent quarks u, d, s, c and b.

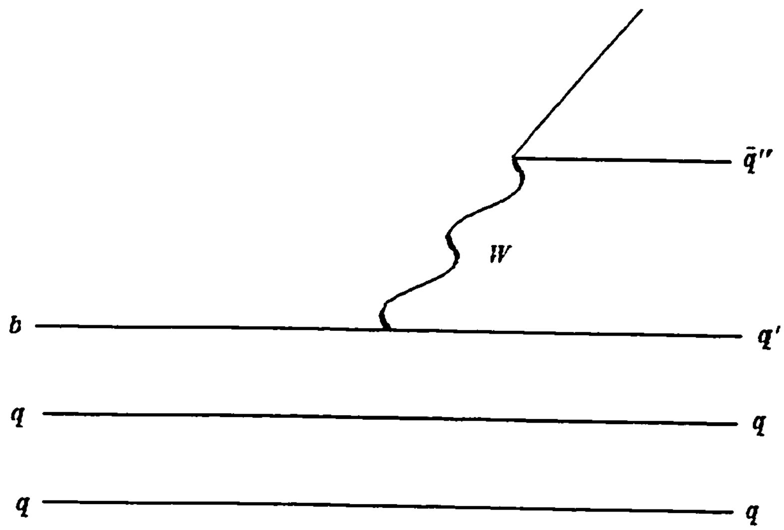

Figure 1.1: External spectator nonleptonic decay mechanism for b-baryons.

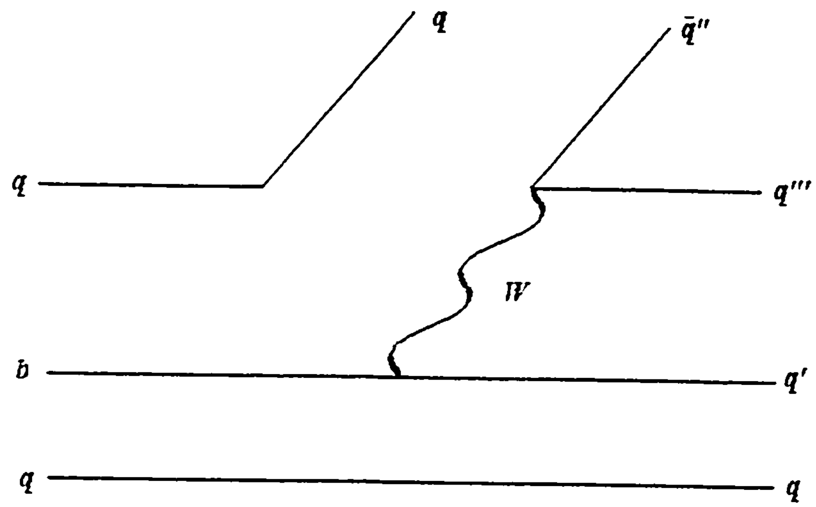

Figure 1.2: Internal spectator nonleptonic decay mechanism (1) for b-baryons.

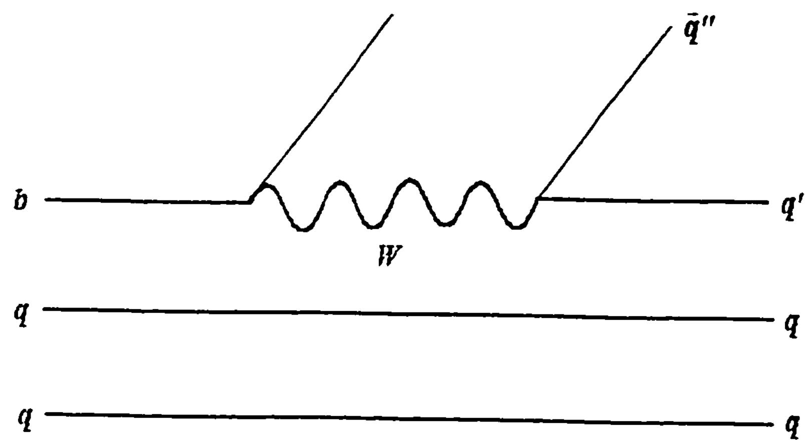

Figure 1.3: Internal spectator nonleptonic decay mechanism (2) for b-baryons.

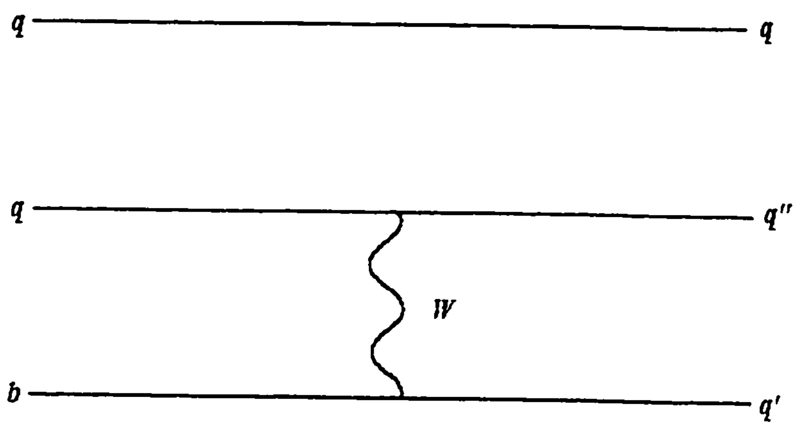

Figure 1.4: W-exchange nonleptonic mechanism for b-baryons.

Chapter 2

Radiative Decays of b-Baryons

In this chapter. we investigate the radiative decays of b-flavored baryons in the non-relativistic naive quark model. The decays are of the form

$$B_i\left(J^P = \frac{3}{2}^+\left(\frac{1}{2}^+\right)\right) \longrightarrow B_f\left(J^P = \frac{1}{2}^+\right) + \gamma\left(l^P = 1^-\right)$$

We assume that the quarks are subject to a two body confining[1] potential $V\left(\mathbf{r}_q, \mathbf{r}_{q'}\right)$ and suppose no specific form for it. We treat the emission of the photon as a first order perturbation to the original Hamiltonian and make necessary approximations when needed. The ultimate goal of this chapter will be the determination of radiative decay widths.

[1] Confinement refers to the seemingly infinite resistance that quarks show when one tries to separate them. Quarks are thought to appear always in bound quark-antiquark states (i.e., mesons) or 3-quark states (i.e., baryons) and never in isolation.

2.1 Theoretical Framework

In the non-relativistic limit, the total Hamiltonian for a bound 3-quark system (i.e., a baryon in our case) is given by

$$H_0 = \sum_q \frac{\mathbf{p}_q^2}{2m_q} + \frac{1}{2}\sum_{q,q'} V\left(\mathbf{r}_q, \mathbf{r}_{q'}\right) \tag{2.1}$$

where $q, q' = q_1, q_2, q_3$; m_q, $\mathbf{r}_q$, and $\mathbf{p}_q = -i\boldsymbol{\nabla}_q$ stand for the mass, the position vector operator, and the linear momentum vector operator of quark q, respectively.

We can rewrite H_0 in terms of the Pauli matrices as follows

$$H_0 = \sum_q \frac{\left(\boldsymbol{\sigma}_q \cdot \mathbf{p}_q\right)^2}{2m_q} + \frac{1}{2}\sum_{q,q'} V\left(\mathbf{r}_q, \mathbf{r}_{q'}\right) \tag{2.2}$$

where the subscript in the vector operator $\boldsymbol{\sigma}_q$ indicates that it acts only on the wave function of quark q.

Now, let us introduce the electromagnetic interaction by assuming that the system is plunged into an external electromagnetic field of vector potential $\mathbf{A}(\mathbf{r},t)$, i.e., one makes the gauge invariant *minimal substitution*[2]

$$\mathbf{p}_q \longrightarrow \mathbf{p}_q - Q_q e \mathbf{A}(\mathbf{r}_q, t) \tag{2.3}$$

where $Q_q e$ is the electric charge of the quark q, and e the magnitude of the electron electric charge.

[2] The adjective minimal comes from the fact that this is the simplest gauge invariant prescription which introduces the electromagnetic interaction between charged particles (i.e., quarks in this case) and photons. Although more complex gauge invariant prescriptions could equally be used, the experimantal data suggest that no more than this prescription is actually needed [34].

Adopting the Coulomb gauge[3]

$$\nabla.\mathbf{A} = 0, \tag{2.4}$$

we can show that the perturbed Hamiltonian reads

$$H = H_0 + H^{int}(\alpha) + O(\alpha^2) \tag{2.5}$$

where α is the electromagnetic coupling constant, $\alpha = \frac{e^2}{4\pi}$, $H^{int}(\alpha)$ is the first order perturbation caused by the inclusion of the electromagnetic field, and $O(\alpha^2)$ is a second order perturbation term which we shall neglect. The explicit expression for H^{int} reads

$$H^{int} = \sum_q \frac{-Q_q e}{2m_q}[-i2\mathbf{A}_q.\boldsymbol{\nabla}_q + \sigma_q.(\boldsymbol{\nabla}_q \times \mathbf{A}(\mathbf{r}_q,t))] \tag{2.6}$$

At this stage, it seems appropriate to make the following remarks in order to avoid unnecessary calculations: we notice that:

1) the first term in the above expression is the one that ultimately gives rise to the electric multipole transitions El (electric dipole $E1$ to first approximation, electric quadrupole $E2$ at a higher level and so on [35].

2) In contrast the second term gives rise to the magnetic multipole transitions Ml (magnetic dipole transitions $M1$, magnetic quadrupole transitions $M2$, etc.)

Since, $E1$ and $M2$ transitions are parity-changing transitions (cf. Appendix A) and since we are interested solely in $J_i^+ \to J_f^+$ (i.e., *positive parity initial states* transforming into *positive parity final states)*, we can disregard these transitions. As

[3] The Coulomb gauge is useful when only a pure radiation field (in our case $\mathbf{A}(\mathbf{r},t)$)need be quantized [34].

for the $E2$ transitions, although positive-parity like $M1$ transitions and have even almost the same transition probability (cf. Appendix A) are in fact forbidden in the quark model, in line with the data, as noticed by Becchi and Morpurgo [36]. Hence our decays will proceed mainly via magnetic dipole transitions $M1$.

Under these conditions the interaction Hamiltonian (2.6) reduces to

$$H^{int} = \sum_q \frac{-Q_q e}{2m_q} \sigma_q . \left(\nabla_q \times \mathbf{A}\left(\mathbf{r}_q, t\right)\right) \tag{2.7}$$

Let us now turn to the electromagnetic field $\mathbf{A}(\mathbf{r},t)$. After quantization, the vector potential $\mathbf{A}(\mathbf{r},t)$ becomes the operator [37]

$$\mathbf{A}(\mathbf{r},t) = \frac{1}{\sqrt{2V}} \sum_{\mathbf{k}'} \sum_{\lambda'} \frac{1}{\sqrt{\omega'}} \epsilon^{(\lambda')} \exp\left[-i\left(\omega' t + \mathbf{k}'.\mathbf{r}\right)\right] a_{\lambda'}\left(\mathbf{k}'\right) + \text{H.c.} \tag{2.8}$$

where $V = L^3$ is the volume in which the field $\mathbf{A}(\mathbf{r},t)$ is assumed to be enclosed; $\mathbf{k}$, ω and $\epsilon^{(\lambda)}$ are respectively the momentum, the energy and the polarization vector of a photon γ. Note that (2.8) is expressed as the sum of a term and its Hermitian conjugate to guarantee the reality of $\mathbf{A}(\mathbf{r},t)$. Recall that the momentum and the energy of a photon are related by

$$|\mathbf{k}| = \omega \tag{2.9}$$

As mentioned above, λ stands for the polarization. If one chooses to work with the circular basis, λ takes on symbolic values "+" for positive-helicity photons, and "−" for negative-helicity photons.

In the circular basis one has the following identities

$$
\begin{aligned}
\epsilon^{(\lambda)} &= -\lambda\frac{1}{\sqrt{2}}\left(\epsilon^{(1)}+\lambda i\epsilon^{(2)}\right) && (2.10)\\
\epsilon^{(\lambda)*} &= -\lambda\epsilon^{(-\lambda)} && (2.11)\\
\epsilon^{(\lambda)}.\epsilon^{(\lambda')*} &= \delta_{\lambda\lambda'} && (2.12)\\
\mathbf{k}.\epsilon^{(\lambda)} &= 0 && (2.13)\\
\sum_{\lambda}\epsilon_i^{(\lambda)}.\epsilon_j^{(\lambda)*} &= \delta_{ij}-\frac{k_i k_j}{\mathbf{k}^2} && (2.14)
\end{aligned}
$$

where $\epsilon^{(1)}$ and $\epsilon^{(2)}$are the vectors of the planar basis.

As for $a_\lambda(\mathbf{k})$ and its Hermitian conjugate $a_\lambda^\dagger(\mathbf{k})$, they stand, respectively, for the annihilation and creation operators of a photon with momentum $\mathbf{k}$ and polarization λ. They are defined by

$$
\begin{aligned}
a_{\lambda'}\left(\mathbf{k}'\right)|n_\lambda(\mathbf{k})\rangle &= \delta_{\lambda\lambda'}\delta\left(\mathbf{k}-\mathbf{k}'\right)\sqrt{n_\lambda(\mathbf{k})}\,|n_\lambda(\mathbf{k})-1\rangle\\
a_{\lambda'}^\dagger\left(\mathbf{k}'\right)|n_\lambda(\mathbf{k})\rangle &= \delta_{\lambda\lambda'}\delta\left(\mathbf{k}-\mathbf{k}'\right)\sqrt{n_\lambda(\mathbf{k})+1}\,|n_\lambda(\mathbf{k})+1\rangle\,,
\end{aligned}
$$

and one can verify that they satisfy the following relations

$$
\begin{aligned}
\left[a_\lambda(\mathbf{k})\,,a_\lambda^\dagger(\mathbf{k})\right] &= \delta_{\lambda\lambda'}\delta\left(\mathbf{k}-\mathbf{k}'\right)\\
a_\lambda^\dagger(\mathbf{k})\,a_\lambda(\mathbf{k})\,|n_\lambda(\mathbf{k})\rangle &= n_\lambda(\mathbf{k})\,|n_\lambda(\mathbf{k})\rangle\\
|n_\lambda(\mathbf{k})\rangle &= \frac{1}{\sqrt{n_\lambda(\mathbf{k})}}\left(a_\lambda^\dagger(\mathbf{k})\right)^{n_\lambda(\mathbf{k})}|0\rangle
\end{aligned}
$$

where $|n_\lambda(\mathbf{k})\rangle$ is the quantum mechanical field of n photons with momentum $\mathbf{k}$ and polarization λ.

The probability amplitudes of the decays can be written as

$$H_{fi}^{int} \equiv \langle B_f | H^{int} | B_i \rangle = \frac{i\exp(i\omega t)}{\sqrt{2V\omega}} \sum_q \frac{Q_e e}{2m_q} \langle B_f | \sigma_q . \left(\mathbf{k} \times \epsilon^{(\lambda)*} \right) \exp\left(-i\mathbf{k}.\mathbf{r}_q\right) | B_i \rangle \tag{2.15}$$

These matrix elements are difficult to evaluate due to the presence of the exponential term $\exp(-i\mathbf{k}.\mathbf{r}_q)$. Fortunately, one notices that $k.r_q \geqslant |\mathbf{k}.\mathbf{r}_q| \geqslant 0$,i.e., one can write $\mathbf{k}.\mathbf{r}_q \approx kr_q = \omega r_q$. On the other hand, a typical radial distance for a quark q confined in a baryon is $r_q \approx 10^{-15}$ m while $\omega \approx 100$ MeV$= 10^8$ eV , this yields

$$\mathbf{k}.\mathbf{r}_q \approx 0.05$$

Thus, to within a relative error of 5%, we can consider that

$$\exp(-i\mathbf{k}.\mathbf{r}_q) = 1. \tag{2.16}$$

This is the well known *dipole approximation*[4] which we shall adopt in what follows.

Using the dipole approxiamation, we can rewrite (2.15) as follows

$$H_{fi}^{int} = \frac{i\exp(i\omega t)}{\sqrt{2V\omega}} \sum_q \frac{Q_e e}{2m_q} \langle B_f | \sigma_q . \left(\mathbf{k} \times \epsilon^{(\lambda)*} \right) | B_i \rangle \tag{2.17}$$

or equivalently as

$$H_{fi}^{int} = \frac{i\exp(i\omega t)}{\sqrt{2V\omega}} \sum_q \langle B_f | \mu_{qz} \left(\mathbf{k} \times \epsilon^{(\lambda)*} \right)_z | B_i \rangle$$

[4] Physically speaking, making the dipole approximation simply amounts to assuming that the wavelength of photon is much larger than the size of the system (i.e., the baryon in this case). In other words, we assume that the variation of the electromagnetic field through the baryon is negligible [38].

$$+\frac{i\exp(i\omega t)}{2\sqrt{2V\omega}}\sum_q \langle B_f|\,\mu_{q_+}\left(\mathbf{k}\times\epsilon^{(\lambda)^*}\right)_-|B_i\rangle$$
$$+\frac{i\exp(i\omega t)}{2\sqrt{2V\omega}}\sum_q \langle B_f|\,\mu_{q_-}\left(\mathbf{k}\times\epsilon^{(\lambda)^*}\right)_+|B_i\rangle \tag{2.18}$$

where

$$\mu=\frac{Q_q e}{2m_q}\sigma \tag{2.19}$$

is the magnetic moment operator of a Dirac particle (i.e., an elementary or structureless fermion with gyromagnetic ratio $g=2$)[5] .

The next step is to calculate the decay widths. To proceed we notice that although the decaying particle transforms from a well defined state ($|B_i\rangle$) to a well defined state ($|B_f\rangle$), the total system transforms, in fact, to a continuum of final states provided by the coupling of $|B_f\rangle$ to the many possible energy states of the emitted photon (although the probability of finding this energy outside a small interval centered about ω is negligible). This will allow us to use the Fermi golden rule to calculate the decay widths that we need.

According to the Fermi golden rule, the decay width of a transition characterized by a probability amplitude H_{fi}^{int} is given by

$$d\Gamma_\gamma=2\pi\left|H_{fi}^{int}\right|^2\rho_\gamma(\omega) \tag{2.20}$$

where $\rho_\gamma(\omega)$ is the photon energy density in the interval $[\mathbf{k},\mathbf{k}+d\mathbf{k}]$.

[5] The treatment of constituent quarks (i.e., bare quarks plus the cloud of gluons and quark-antiquark pairs surrounding them) as structureless particles is a famous assumption of the quark model. Its justification lies in its undisputed predictive power

Working in the rest frame of the initial baryon B_i, we have

$$\mathbf{p}_i = 0 = \mathbf{p}_f + \mathbf{k} \tag{2.21}$$

$$E_i = m_{B_i} = E_f + \omega \tag{2.22}$$

$$E_f = \sqrt{m_{B_f}^2 + \mathbf{p}_f^2} \tag{2.23}$$

On the other hand, when treated non-relativistically in the phase space, the number of photon states in the interval $[\mathbf{k}, \mathbf{k}+d\mathbf{k}]$ is given by

$$\begin{aligned} dN &= \frac{V}{(2\pi)^3} d\mathbf{k} \\ &= \frac{V}{(2\pi)^3} \frac{d\mathbf{k}}{d\omega} d\omega \\ &= \frac{V}{(2\pi)^3} \omega^2 d\omega d\Omega \end{aligned}$$

and, hence, the density is

$$\begin{aligned} \rho_\gamma(\omega) &= d\Omega \frac{V}{(2\pi)^3} \int_0^\infty \omega^2 \delta(E_i - E_f - \omega)\, d\omega \\ &= d\Omega \frac{V}{(2\pi)^3} \omega^2 (1 - \frac{\omega}{m_{B_i}}) \end{aligned} \tag{2.24}$$

but for bottom baryons $m_{B_i} \approx 6 \times 10^3$ MeV, i.e.,

$$\frac{\omega}{m_{B_i}} \approx 0.01 \tag{2.25}$$

one has as a result

$$\rho_\gamma(\omega) = d\Omega \frac{V}{(2\pi)^3} \omega^2 \tag{2.26}$$

Substituting (2.26) into (2.20) we obtain

$$d\Gamma_\gamma = \frac{V\omega^2}{4\pi^2} \left|H^{int}_{fi}\right|^2 d\Omega \tag{2.27}$$

In order to calculate Γ_γ one does not only integrate over all spatial direction, but also sums over all B_f spin projections and averages over all B_i spin projections. That is

$$\begin{aligned}
\Gamma_\gamma &= \frac{V\omega^2}{4\pi^2}\frac{1}{2J_i+1}\sum_{J_{iz}}\sum_{J_{fz}}\sum_{\lambda}\int_{all\ space} \left|H^{int}_{fi}\right|^2 d\Omega \\
&= \frac{3\omega}{8\pi^2}\frac{2}{2J_i+1}\sum_q \left|\langle B_f|\,\mu_{qz}\,|B_i\rangle\right|^2 \int_{all\ space}\sum_\lambda \left|\left(\mathbf{k}\times\epsilon^{(\lambda)*}\right)\right|^2 d\Omega
\end{aligned} \tag{2.28}$$

which gives after integration

$$\Gamma_\gamma = \omega^3 \frac{2}{2J_i+1}\left|\langle B_f|\,\mu_z\,|B_i\rangle\right|^2 \tag{2.29}$$

where $\mu_z \equiv \sum_q \mu_{qz}$, is called *the static magnetic moment operator* of the baryon[6] . For convenience, one expresses the transition magnetic dipole moment $\langle B_f|\,\mu_z\,|B_i\rangle$ in units of the nuclear magneton $\mu_N \equiv \frac{e}{2m_p}$, where m_p is the mass of the proton. By doing so, one obtains the final expression for the radiative decay widths, viz.,

$$\Gamma_\gamma = \alpha\frac{\omega^3}{m_p^2}\frac{2}{2J_i+1}\mu^2 \tag{2.30}$$

where the energy of the emitted photon is given in terms of the initial and final masses by

[6] In the naive (static) quark model the baryons are envisioned as consisting of quarks all in a relative S-wave state. Thus the baryon magnetic moment is the mere sum of its constituent quarks magnetic moments.

$$\omega = \frac{m_{B_i}^2 - m_{B_f}^2}{2m_{B_i}} \tag{2.31}$$

2.2 Applications

In this section we apply the results of the previous section to the determination of the radiative decay widths of the b-flavored baryons given in Table 2.1. Here we assume as in [39] that the light diquark of bottom baryons belong either to the antitriplet representation $\mathbf{3}^*$ of $SU(3)$ or to its sextet representation $\mathbf{6}$.

Therefore the flavor-spin wave functions of $J^P = \frac{1}{2}^+$ baryons can be written in compact form as [39]

$$MA_{ij} = \frac{1}{\sqrt{2}}(q_i q_j - q_j q_i) b \chi_{MA} \tag{2.32}$$

$$MS_{ij} = \frac{1}{\sqrt{2}}(q_i q_j + q_j q_i) b \chi_{MS} \tag{2.33}$$

where $i, j = 1, 2, 3$ $(q_1 = u, q_2 = d, q_3 = s)$ and the mixed anti-symmetry (MA) and mixed symmetry (MS) spin wave functions are given by

$$\chi_{MA}^{1/2,1/2} = \tfrac{1}{\sqrt{2}} \left|(\uparrow\downarrow - \downarrow\uparrow) \uparrow\right\rangle; \quad \chi_{MA}^{1/2,-1/2} = \tfrac{1}{\sqrt{2}} \left|(\uparrow\downarrow - \downarrow\uparrow) \downarrow\right\rangle \tag{2.34}$$

$$\chi_{MS}^{1/2,1/2} = \tfrac{1}{\sqrt{6}} \left|2 \uparrow\uparrow\downarrow - (\uparrow\downarrow + \downarrow\uparrow) \uparrow\right\rangle; \quad \chi_{MS}^{1/2,-1/2} = \tfrac{-1}{\sqrt{6}} \left|2 \downarrow\downarrow\uparrow - (\uparrow\downarrow + \downarrow\uparrow) \downarrow\right\rangle \tag{2.35}$$

For $J^P = \frac{3}{2}^+$ bottom baryons one has similar expressions, viz.,

$$S_{ij} = \frac{1}{\sqrt{2}}(q_i q_j + q_j q_i) b \chi_S \tag{2.36}$$

where the symmetric (S) spin wave functions χ_S are given by

$$\chi_S^{3/2,3/2} = |\uparrow\uparrow\uparrow\rangle ; \quad \chi_S^{3/2,-3/2} = |\downarrow\downarrow\downarrow\rangle$$

$$\chi_S^{3/2,1/2} = \frac{1}{\sqrt{3}}|\uparrow\uparrow\downarrow + (\uparrow\downarrow + \downarrow\uparrow)\uparrow\rangle ; \quad \chi_S^{3/2,-1/2} = \frac{1}{\sqrt{3}}|\downarrow\downarrow\uparrow + (\uparrow\downarrow + \downarrow\uparrow)\downarrow\rangle$$

For the numerical calculations one also needs the masses of the bottom baryons together with the magnetic moments of the u. d. s and b quarks .

The value of the s quark magnetic moment deserves a comment. Instead of the naive estimate $\mu_s = \mu_\Lambda = -0.613\ \mu_N$ we choose to adopt a value suggested by *configuration mixing* [7] . The magnetic moment of the b-quark. however. has not been measured. it needs to be estimated. In fact the magnetic moment of a quark q can be expressed in nuclear magnetons as follows

$$\mu_q = \frac{m_p}{m_q} Q_q \mu_N$$

and hence

$$\frac{\mu_b}{\mu_d} = \frac{m_d}{m_b} \simeq \frac{1}{15}$$

In what follows we use the estimate

[7] Configuration mixing attributes the deviation of calculated magnetic moments from measured values to small admixtures of a d-wave contribution to the orbital angular momentum [40].

$$\mu_b = -0.064\ \mu_N \tag{2.37}$$

In Table 2.1 we give the masses of the b-baryons and their wave functions. For the masses, we have used two sets determined by two different methods. The first set was determined using the Feynman-Hellmann theorem and semiempirical mass formulas [41]. As for the second set, it was determined using *SU(3)* and the non-relativistic quark model [39]. The major point of the authors of [41] is that one can exploit regularities in the pattern of known hadron masses to obtain estimates of the masses of unknown hadrons. They managed to give a prediction for the mass of the Λ_b baryon (which is the only bottom baryon mass known experimentally) in good agreement with the experimental value [42]: $m_{\Lambda_b} = 5623 \pm 5 \pm 4$ MeV . In [39] on the other hand $m_{\Lambda_b} = 5623$ MeV is used as an imput parameter.

The values of magnetic moments of u, d, s and b quarks used in our calculations are given in Table 2.2. The numerical results for the decay widths are summarized in Table 2.3.

2.3 Discussion

Unfortunately, no experimental data concerning the decays investigated in this study have, as yet, been reported, although various groups have been anouncing their arrival for a while now. We hope that some measurements will be available soon.

However, as mentioned above (cf. section 1.1), the theoretical estimates of the nonrelativistic quark model are in good agreement with the experimental values. It is therefore reasonable to assume that the present predictions of the radiative decays based on the nonrelativistic quark model are reliable. Particularly, since the dominant decay mode for the baryons $\Xi_b^{0\prime}$, $\Xi_b^{-\prime}$, and Ω_b^* is the radiative mode, we obtain for these states the mean life-times ($\tau = \hbar/\Gamma_\gamma$) :

$$
\begin{aligned}
\tau\left(\Xi_b^{0\prime}\right) &= 1.36 \times 10^{-20} s \\
\tau\left(\Xi_b^{-\prime}\right) &= 1.57 \times 10^{-18} s \\
\tau\left(\Omega_b^{*-}\right) &= 1.65 \times 10^{-16} s
\end{aligned}
$$

Baryons	*Mass* $\pm$ *50 (MeV)* [41]	*Mass (MeV)* [39]	*Wave function*
Λ_b^0	5620	5623 (input)	MA_{12}
Ξ_b^0, Ξ_b^-	5810	5806	$-MA_{13}, MA_{23}$
$\Sigma_b^+, \Sigma_b^0, \Sigma_b^-$	5820	5822	$\frac{1}{\sqrt{2}}MS_{11}, MS_{12}, \frac{1}{\sqrt{2}}MS_{22}$
$\Xi_b^{\prime 0}, \Xi_b^{\prime -}$	5950	5945	MS_{13}, MS_{23}
Ω_b^-	6060	6059	$\frac{1}{\sqrt{2}}MS_{33}$
$\Sigma_b^{*+}, \Sigma_b^{*0}, \Sigma_b^{*-}$	5850	5842	$\frac{1}{\sqrt{2}}S_{11}, S_{12}, \frac{1}{\sqrt{2}}S_{22}$
Ξ_b^{*0}, Ξ_b^{*-}	5980	5962	S_{13}, S_{23}
Ω_b^{*-}	6090	6073	$\frac{1}{\sqrt{2}}S_{33}$

Table 2.1: Masses and wave functions of bottom baryons.

Quark	u	d	s	b
μ_q	1.851	-0.971	-0.73	-0.064

Table 2.2: Values of magnetic moments of quark flavors u, d, s and b in nuclear magnetons.

Radiative Transition	μ	$\Gamma_\gamma[41]$ *(keV)*	$\Gamma_\gamma[39]$ *(keV)*
$\Sigma_b^0 \longrightarrow \Lambda_b^0$	$-\frac{1}{\sqrt{3}}(\mu_u - \mu_d)$	167.15	164.71
$\Sigma_b^{*0} \longrightarrow \Sigma_b^0$	$\frac{\sqrt{2}}{3}(\mu_u + \mu_d - 2\mu_b)$	0.02	0.01
$\Sigma_b^{*0} \longrightarrow \Lambda_b^0$	$\sqrt{\frac{2}{3}}(\mu_u - \mu_d)$	252.31	218.42
$\Sigma_b^{*+} \longrightarrow \Sigma_b^+$	$\frac{2\sqrt{2}}{3}(\mu_u - \mu_b)$	0.36	0.11
$\Sigma_b^{*-} \longrightarrow \Sigma_b^-$	$\frac{2\sqrt{2}}{3}(\mu_d - \mu_b)$	0.08	0.02
$\Xi_b^{\prime 0} \longrightarrow \Xi_b^0$	$-\frac{1}{\sqrt{3}}(\mu_u - \mu_s)$	48.75	47.73
$\Xi_b^{\prime -} \longrightarrow \Xi_b^-$	$-\frac{1}{\sqrt{3}}(\mu_d - \mu_s)$	0.42	0.42
$\Xi_b^{*0} \longrightarrow \Xi_b^0$	$\sqrt{\frac{2}{3}}(\mu_u - \mu_s)$	86.65	67.19
$\Xi_b^{*0} \longrightarrow \Xi_b^{\prime 0}$	$\frac{\sqrt{2}}{3}(\mu_u + \mu_s - 2\mu_b)$	0.03	0.01
$\Xi_b^{*-} \longrightarrow \Xi_b^-$	$\sqrt{\frac{2}{3}}(\mu_d - \mu_s)$	0.75	0.58
$\Xi_b^{*-} \longrightarrow \Xi_b^{\prime -}$	$\frac{\sqrt{2}}{3}(\mu_d + \mu_s - 2\mu_b)$	0.06	0.01
$\Omega_b^{*-} \longrightarrow \Omega_b^-$	$\frac{2\sqrt{2}}{3}(\mu_s - \mu_b)$	0.04	0.004

Table 2.3: Transition magnetic moments and radiative decay widths of bottom baryons.

Chapter 3

Baryonic Form Factors

In this chapter, we introduce the general framework for the study of two-body non-leptonic decays of the form

$$B_b(p) \to B(p') + X(q)$$

with $\Delta b = 1$, where B_b is a baryon containing a single bottom quark and B is either a singly charmed baryon or an ordinary hyperon. Both B_b and B are spin-$\frac{1}{2}^+$ baryons; X is either a pseudoscalar meson P ($J^P = 0^-$), a vector meson V ($J^P = 1^-$) or an axial vector meson A ($J^P = 1^+$). We start by writing the hadronic current matrix elements which result from factorization in terms of Pauli spinors and then derive explicit expressions for the form factors. Afterwards, we evaluate the overlap term which contributes to the transition matrix elements. As for the terms whose calculation depends explicitly on the details of the processes, we will treat them in chapter 4 where specific decays will be considered.

3.1 Introduction

To lowest order in the weak interaction, the nonleptonic Hamiltonian can effectively be written in the form

$$H_{eff} = \frac{G_F}{\sqrt{2}} J_\mu J_\mu^\dagger + H.c. \tag{3.1}$$

where

$$G_F = 1.16639 \times 10^{-5}\ GeV^{-2}$$

is the universal weak coupling constant (*Fermi's constant*). The weak current J_μ is given by

$$J_\mu = (\overline{u}\ \overline{c}\ \overline{t})\ \gamma_\mu (1 + \gamma_5) \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}. \tag{3.2}$$

The weak eigenstates d', s', and b' are related to the mass eigenstates d, s, and b through the unitary Cabbibo-Kobayashi-Maskawa (*CKM*) mixing matrix V :

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}. \tag{3.3}$$

It is possible to choose phases for the quark fields so that the matrix V is written as [17]

$$V = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2s_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + s_2c_3e^{i\delta} & c_1s_2s_3 - c_2s_3e^{i\delta} \end{pmatrix}, \tag{3.4}$$

involving 7 parameters , c_1, c_2, c_3, s_1, s_2, s_3 and δ where $c_i \equiv \cos\theta_i$ and $s_i \equiv \sin\theta_i$. The real Euler angles θ_i, $i = 1, 2, 3$ are chosen to lie in the first quadrant such that their sines and cosines are positive. The angle δ may vary in the range $0 \leqslant \delta \leqslant 2\pi$, although the measurements of CP violation in kaon decays force δ to be in the range $0 \leqslant \delta \leqslant \pi$. Hence 4 independent parameters arise in Eq. (3.4) θ_1, θ_2, θ_3, and δ. The unitarity of the CKM matrix provides us with several relations of which [35]

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \tag{3.5}$$

is the most useful.

The effective approach known as the *operator product expansion* (OPE) is the standard framework for the study of nonleptonic bottom baryon decays. It effectively takes into consideration the QCD radiative corrections and permits a separation between the long-distance (i.e., low-energy) contributions of the strong interaction and its short-distance (i.e., high-energy) contributions. In the OPE the probability amplitude for a process such as the nonleptonic weak decay $baryon \rightarrow baryon' + X$

is written as

$$A = \langle X\ baryon' \,|H_{eff}|\, baryon\rangle = \sum_i C_i \,\langle X\ baryon' \,|O_i|\, baryon\rangle$$

where $O_i \equiv O_i\,(\mu)$ are the operators of the effective Hamiltonian, and $C_i \equiv C_i\,(\mu)$ are the *Wilson coefficients*. The effective operators contain all the long-distance QCD contributions and the Wilson coefficients contain all the short-distance QCD contributions. Both O_i and C_i depend on the renormalization scale μ which has the dimensions of energy and separates the two realms of low-energy QCD effects and high-energy QCD effects. Typically μ is chosen to be of the order of few GeV for the decays of mesons containing one heavy quark (i.e., B and D mesons) [43].

In order to calculate the hadronic matrix elements $\langle X\ baryon' \,|O_i|\, baryon\rangle$ we resort to the factorization approximation, since at present such matrix elements are not calculable from first principles. Thus $\langle X\ baryon' \,|O_i|\, baryon\rangle$ reduce to $\langle\ baryon' \,|J_\mu|\, baryon\rangle$ and $\langle 0 \,|J'_\mu|\, X\rangle$. It is the neglect of these final state interactions *(FSI)* which introduces the *color-matching* free parameter $\xi \equiv \frac{1}{N_c}$ (which is assumed to compensate for the neglect of color octet-octet contribution in evaluating the hadronic matrix elements).

The scattering matrix (*S-matrix*) elements of the process $B_b(p) \rightarrow B(p') + X(q)$ are given by

$$\begin{aligned} S_{fi} &\equiv S\,(i \rightarrow f) \\ &\equiv S\,(B_b(p) \rightarrow B(p') + X(q)) \\ &\equiv \langle f_{outgoing} \mid i_{incoming}\rangle \end{aligned}$$

$$\equiv \lim_{\substack{t_0 \to -\infty \\ t \to \infty}} \left\langle f_{free} \left| P \exp\left(-i \int_{t_0}^{t} H_{int}\, dt \right) \right| i_{free} \right\rangle \tag{3.6}$$

where

$$H_{int} \equiv H_s + H_{em} + H_w$$

H_s being the strong interaction Hamiltonian; H_{em} the electromagnetic interaction Hamiltonian; and H_w the weak interaction Hamiltonian which in our case reduces to the weak nonleptonic Hamiltonian (3.1). P is the *Dyson chronological (*or *time-ordering) product operator* defined by

$$P\left(V\left(t_\alpha\right), \ldots, V\left(t_\varrho\right)\right) \equiv V\left(t_1\right) \times \ldots \times V\left(t_n\right)$$

where $t_1 \geqslant \ldots \geqslant t_n$.

To first order in the weak interaction, Eq. (3.6) gives as the transition matrix (*T-matrix*) element

$$T_{fi} = -i \left\langle f \left| \int_{-\infty}^{\infty} H_{eff} dt \right| i \right\rangle \tag{3.7}$$

where $|i\rangle$ and $|f\rangle$ are now the eigenstates of the strong and electromagnetic Hamiltonians. T_{fi} can be written as [44]

$$T_{fi} = i\,(2\pi)^4\, \delta^4(p - (p' + q))\widetilde{T}_{fi} \tag{3.8}$$

where the *reduced T-matrix element* $\widetilde{T}_{fi}$ reads

$$\widetilde{T}_{fi} = i\frac{G'}{\sqrt{2}} \left(\frac{1}{2\pi}\right)^3 \sqrt{\frac{mm'}{EE'}} \left\langle 0 \left| J'_\mu \right| X(q) \right\rangle \left\langle B(p') \left| J_\mu \right| B_b(p) \right\rangle \tag{3.9}$$

The constant $G' \propto G_F$ and its detailed structure depends on the specific decay at hand. The spin-$\frac{1}{2}^+$ baryons are characterized by the field operators

$$\psi(x) = \left(\frac{1}{2\pi}\right)^{3/2} \int d\mathbf{k} \sqrt{\frac{m}{k_0}} \sum_{r=\uparrow,\downarrow} a_r(k)\, u_r(k) \exp(ikx)$$

where $u_r(p)$ are the free particle Dirac spinors given in the nonrelativistic limit by

$$u_{\uparrow,\downarrow}(p) = \sqrt{\frac{E+m}{2E}} \begin{pmatrix} 1 \\ \frac{\sigma.\mathbf{p}}{E+m} \end{pmatrix} \chi_{\uparrow,\downarrow} \tag{3.10}$$

E, $\mathbf{p}$, and m being respectively the total energy, the momentum and the mass of the baryon related by the identity

$$E = \sqrt{\mathbf{p}^2 + m^2} \tag{3.11}$$

$\chi_{\uparrow,\downarrow}$ are the Pauli spinors expressed as

$$\chi_\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix}; \quad \chi_\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{3.12}$$

and $a_r(k)$ the annihilation operator defined by

$$\begin{aligned}
a_r(k)\,|n_{r'}(p)\rangle &\equiv \delta_{rr'}\delta^4(p-k)\,|n_{r'}(p)-1\rangle \\
\left\{a_r(k)\,, a^\dagger_{r'}(k')\right\} &\equiv \delta_{rr'}\delta^4(p-k) \\
\{a_r(k)\,, a_{r'}(k')\} &\equiv 0
\end{aligned}$$

The hadronic current matrix element

$$\langle 0\,|J'_\mu|\,X(q)\rangle = \begin{cases} \langle 0\,|J'_\mu|\,V(q)\rangle = \langle V(q)\,|J'_\mu|\,0\rangle = F_V \epsilon_\mu \\ \text{or} \\ \langle 0\,|J'_\mu|\,P(q)\rangle = -\langle P(q)\,|J'_\mu|\,0\rangle = F_P q_\mu \end{cases} \tag{3.13}$$

depending on whether X is a vector meson V or a pseudoscalar meson P. F_V and F_P are respectively the decay constants of the V meson and the P meson; ϵ_μ is the polarization 4-vector of the V meson and $q_\mu = p_\mu - p'_\mu$ is the momentum transfer 4-vector.

To maximize parity violation, *$V-A$ theory* stipulates that J_μ consist of the sum of a vector part V_μ and an axial vector part A_μ, i.e.,

$$J_\mu = V_\mu + A_\mu \tag{3.14}$$

Since B_b and B are both single baryon states possessing even relative parity, Lorentz invariance yields for $\langle B(p')\,|J_\mu|\,B_b(p)\rangle$ the general form[1]

$$\begin{aligned}\langle B(p')\,|J_\mu|\,B_b(p)\rangle &= \langle B(p')\,|(V_\mu + A_\mu)|\,B_b(p)\rangle \\ &= \overline{u}(p')\Gamma_\mu u(p) \\ &\equiv i\overline{u}(p')\{[g_V\,(s)\,\gamma_\mu + ih_V\,(s)\,q_\mu \\ &\quad +f_V\,(s)\,\sigma_{\mu\nu}q_\nu] \\ &\quad +[g_A(s)\gamma_\mu\gamma_5 + h_A(s)\gamma_5\sigma_{\mu\nu}q_\nu \\ &\quad -if_A(s)\gamma_5 q_\mu]\}u(p)\end{aligned} \tag{3.15}$$

where

$$\overline{u}(p') \equiv u^\dagger\,(p')\,\gamma_4 = \sqrt{\frac{E'+m'}{2E'}}\begin{pmatrix} 1, & -\frac{\sigma.p'}{E'+m'}\end{pmatrix}\chi^\dagger_{\uparrow,\downarrow}$$

[1] Although relativistic invariance alone would allow scalar, pseudoscalar, vector, axial vector, and tensor interaction terms to contribute to J_μ, it is an empirical fact that only left-handed leptons participate in the charged current weak interactions, this restricts the possibilities to the combination $V-A$ (vector minus axial vector) and is reponsible for the presence of the factor (scalar−scalar$\cdot\gamma_5$) noted in (3.16). The difference in sign between the historical name $V-A$ and (3.14) is conventional; it is due to the definition of the Dirac γ−matrices that is adopted in this study (cf. Appendix B).

g_V, f_V, h_V, g_A, f_A, and h_A; known as the *weak current form factors*, are simply scalar functions of $s = -q^2$.

3.2 Reduction of Hadronic Matrix Elements

Here we reduce the matrix element $\langle B(p')\,|J_\mu|\,B_b(p)\rangle$ from Dirac spinors to Pauli spinors.

$$\begin{aligned}\langle B(p')\,|J_\mu|\,B_b(p)\rangle &= \overline{u}(p')\Gamma_\mu u(p)\\ &\equiv i\overline{u}(p')\{(g_V(s) - g_A(s)\gamma_5)\,\gamma_\mu\\ &\quad + (f_V(s) + h_A(s)\gamma_5)\,\sigma_{\mu\nu}q_\nu\\ &\quad + i\,(h_V(s) - f_A(s)\gamma_5)\,q_\mu\}u(p) \qquad (3.16)\end{aligned}$$

therefore

$$\begin{aligned}\langle B(p')\,|J_4|\,B_b(p)\rangle &= iu^\dagger(p')\gamma_4[(g_V(s) - g_A(s)\gamma_5)\,\gamma_4\\ &\quad + (f_V(s) - h_A(s)\gamma_5)\,\sigma_{4\nu}q_\nu\\ &\quad + i\,(h_V(s) - f_A(s)\gamma_5)\,q_4]u(p) \qquad (3.17)\end{aligned}$$

In what follows, it is to be understood that the current components J_μ are sandwiched between $\chi_{\uparrow,\downarrow}$ and $\chi^\dagger_{\uparrow,\downarrow}$.

The Dirac γ-matrices and some of their properties encountered in this calculation are given in Appendix B. We choose to work in the rest frame of the bottom baryon, i.e., we take

$$\mathbf{p} = 0 \Rightarrow \mathbf{p}' = -\mathbf{q} \qquad (3.18)$$

Thus, the evaluation of the matrix element $\langle B(p')|J_4|B_b(p)\rangle$ yields

$$\begin{aligned}
\langle B(p')|J_4|B_b(p)\rangle &= \sqrt{\frac{E'+m'}{2E'}}\{ig_V(s) - h_V(s)q_4 - if_V(s)\frac{\mathbf{q}^2}{E'+m'} \\
&+ \frac{\boldsymbol{\sigma}.\mathbf{q}}{E'+m'}[ig_A(s) - f_A(s)q_4 \\
&+ ih_A(s)(E'+m')]\}
\end{aligned} \tag{3.19}$$

If one uses the identity $J_4 = iJ_0$, one obtains

$$\begin{aligned}
\langle B(p')|J_0|B_b(p)\rangle &= \sqrt{\frac{E'+m'}{2E'}}\{g_V(s) - h_V(s)q_0 - f_V(s)\frac{\mathbf{q}^2}{E'+m'} \\
&+ \frac{\boldsymbol{\sigma}.\mathbf{q}}{E'+m'}[g_A(s) - f_A(s)q_0 \\
&+ h_A(s)(E'+m')]\}
\end{aligned} \tag{3.20}$$

Similarly, one obtains for the spatial part of the current matrix elements $\langle B(p')|\mathbf{J}|B_b(p)\rangle$

$$\begin{aligned}
\langle B(p')|\mathbf{J}|B_b(p)\rangle &= \sqrt{\frac{E'+m'}{2E'}}\{\left(h_A(s)q_0 + h_A(s)\frac{\mathbf{q}^2}{E'+m'} - g_A(s)\right)\boldsymbol{\sigma} \\
&- \left(h_V(s) + \frac{1}{E'+m'}(g_V(s) + f_V(s)q_0)\right)\mathbf{q} \\
&- \frac{1}{E'+m'}(h_A(s) + f_A(s))\,\mathbf{q}.(\boldsymbol{\sigma}.\mathbf{q}) \\
&- i\left(\frac{1}{E'+m'}(g_V(s) + f_V(s)q_0) + f_V(s)\right)\boldsymbol{\sigma}\times\mathbf{q}\}
\end{aligned} \tag{3.21}$$

Now we reduce the quark level current $j_\mu = i\bar{q}\gamma_\mu(1+\gamma_5)b$. Considering that $p_q = p_3'$, $p_b = p_3$ one gets

$$\begin{aligned}
j_0 &= \frac{1}{2\sqrt{E_3E_3'(E_3+m_3)(E_3'+m_3')}}\{(E_3+m_3)(E_3'+m_3') \\
&- (E_3'+m_3')(\boldsymbol{\sigma}.\mathbf{p}_3) - (E_3+m_3)(\boldsymbol{\sigma}.\mathbf{p}_3') \\
&+ \mathbf{p}_3'.\mathbf{p}_3 + i\boldsymbol{\sigma}.(\mathbf{p}_3'\times\mathbf{p}_3)\}
\end{aligned} \tag{3.22}$$

Similarly one obtains for the spatial part of the current

$$\begin{aligned} \mathbf{j} &= \frac{1}{2\sqrt{E_3E_3'(E_3+m_3)(E_3'+m_3')}}\{[(E_3+m_3)(E_3'+m_3')+\mathbf{p}_3'.\mathbf{p}_3]\boldsymbol{\sigma} \\ &+(E_3'+m_3')\mathbf{p}_3+(E_3+m_3)\mathbf{p}_3'+i(E_3+m_3)(\boldsymbol{\sigma}\times\mathbf{p}_3') \\ &-i(E_3'+m_3')(\boldsymbol{\sigma}\times\mathbf{p}_3)+i\mathbf{p}_3'\times\mathbf{p}_3-(\boldsymbol{\sigma}.\mathbf{p}_3)\mathbf{p}_3'-(\boldsymbol{\sigma}.\mathbf{p}_3')\mathbf{p}_3\} \end{aligned} \tag{3.23}$$

If now we consider that the heavy b-quark is stationary in the rest reference frame of the b-baryon, i.e.,

$$|\mathbf{p}_3| = 0 \tag{3.24}$$

then

$$E_3 = m_3 \tag{3.25}$$

$$E_3' = \sqrt{\mathbf{q}^2+m_3'^2} \tag{3.26}$$

and hence

$$j_0 = \frac{1}{\sqrt{2E_3'(E_3'+m_3')}}[E_3'+m_3'-\boldsymbol{\sigma}.\mathbf{p}_3'] \tag{3.27}$$

$$\mathbf{j} = \frac{-1}{\sqrt{2E_3'(E_3'+m_3')}}[(E_3'+m_3')\boldsymbol{\sigma}+\mathbf{q}+i\boldsymbol{\sigma}\times\mathbf{q}] \tag{3.28}$$

3.3 Form Factors

The state function for a baryon B may be written as

$$|B\rangle = |color\rangle_A\,|space,\ spin,\ flavor\rangle_S\ ,$$

where the subscripts S and A indicate symmetry or antisymmetry under interchange of any two equal mass quarks. At the quark level the evaluation of the hadronic matrix element $\langle B(p')\,|J_\mu|\,B_b(p)\rangle$ involves terms of the type

$$\langle color'|color\rangle = 1 \tag{3.29}$$

$$\langle space'|\delta(\mathbf{r}_1-\mathbf{r}_2)|\,space\rangle \equiv I \tag{3.30}$$

$$\left\langle spin.\ flavor'\left|a^\dagger_{q_3'}a_b\right|spin,\ flavor\right\rangle \equiv \xi_V \tag{3.31}$$

$$\left\langle spin.\ flavor'\left|a^\dagger_{q_3'}a_b(\sigma_3)_b\right|spin.\ flavor\right\rangle \equiv \xi_A, \tag{3.32}$$

where I is the overlap integral. and ξ_V and ξ_A are the spin-flavor part of the matrix elements.

Treating the Pauli matrices as elements of a vector space. one obtains from (3.20) and (3.27)

$$g_V(s) - h_V(s)\,q_0 - f_V(s)\,\frac{\mathbf{q}^2}{E'+m'} = I\xi_V\sqrt{\frac{E'(E_3'+m_3')}{E_3'(E'+m')}}. \tag{3.33}$$

and

$$g_A(s) - f_A(s)\,q_0 - h_A(s)\,(E'+m') = I\xi_A\sqrt{\frac{E'(E'+m')}{E_3'(E_3'+m_3')}}. \tag{3.34}$$

From (3.23) and (3.28) one gets

$$g_A(s) - h_A(s)\left(q_0+\frac{\mathbf{q}^2}{E'+m'}\right) = I\xi_A\sqrt{\frac{E'(E_3'+m_3')}{E_3'(E'+m')}}. \tag{3.35}$$

$$g_V(s) + h_V(s)\,(E'+m') + f_V(s)\,q_0 = I\xi_V\sqrt{\frac{E'(E'+m')}{E_3'(E_3'+m_3')}}. \tag{3.36}$$

$$g_V(s) + f_V(s)\,(E'+m'+q_0) = I\xi_V\sqrt{\frac{E'(E'+m')}{E_3'(E_3'+m_3')}}. \tag{3.37}$$

and finally

$$h_A(s) + f_A(s) = 0 \tag{3.38}$$

If in addition, one uses the fact that

$$q = p - p' \Leftrightarrow \begin{cases} \mathbf{q} = \mathbf{p} - \mathbf{p}' \\ \& \\ q_0 = p_0 - p_0' = E - E' \end{cases}$$

and recalls that we have chosen to work in the rest frame of B_b, i.e., in which $\mathbf{p} = \mathbf{0}$, then one obtains

$$q_0 = m - E' \; \& \; \mathbf{q} = -\mathbf{p}' \Leftrightarrow \mathbf{q}^2 = E'^2 - m'^2 \tag{3.39}$$

Inserting (3.39) into Eqs. (3.33)-(3.38), one can write

$$g_V(s) = \frac{I}{m}\xi_V a(E', E_3'); \quad g_A(s) = \frac{I}{m}\xi_A a(E', E_3') \tag{3.40}$$

$$h_A(s) = -f_A(s); \quad h_A(s) = \frac{I}{m}\xi_A b(E', E_3') \tag{3.41}$$

$$h_V(s) = f_V(s); \quad h_V(s) = \frac{I}{m}\xi_V b(E', E_3') \tag{3.42}$$

where

$$a(E', E_3') = \frac{1}{2}\frac{(E'+m')\left(1-\frac{m'}{m}\right) + (E_3'+m_3')\left(1+\frac{m'}{m}\right)}{\sqrt{(E'+m')(E_3'+m_3')}}\sqrt{\frac{E'}{E_3'}} \tag{3.43}$$

$$b(E', E_3') = \frac{1}{2}\frac{(E'+m') - (E_3'+m_3')}{\sqrt{(E'+m')(E_3'+m_3')}}\sqrt{\frac{E'}{E_3'}}. \tag{3.44}$$

In case B_b belongs to the antitriplet representation **3*** of SU(3), one can show that $\xi_V = \xi_A$ (see chapter 4), and one recovers the relations derived from the heavy quark spin symmetry limit [45], i.e..

$$\begin{aligned} g_A(s) &= g_V(s) \equiv f_1 \\ h_V(s) &= f_V(s) = h_A(s) = -f_A(s) \equiv \frac{1}{m} f_2. \end{aligned} \tag{3.45}$$

3.4 Calculation of the Overlap Integral

The largest interaction elements come from the interaction between constituent quarks giving an amplitude proportional to the square of the wave function when two quarks are at the same point [46]. If $\psi_S(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$ is a baryon spatial wave function in the r-representation, then its counterpart in the p-representation is given by the Fourier transform

$$\begin{aligned} \widetilde{\psi}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) &= \mathcal{N} \int \psi_S(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) \exp\left[i\left(\mathbf{p}_1\mathbf{r}_1 + \mathbf{p}_2\mathbf{r}_2 + \mathbf{p}_3\mathbf{r}_3\right)\right] \\ &\quad \times \delta(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3)\ d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3 \end{aligned} \tag{3.46}$$

where $\mathcal{N}$ is a normalization constant. The overlap integral is then given in terms of the normalized wave functions by [46]

$$\begin{aligned} I &= \int \widetilde{\psi}^{\dagger}(\mathbf{p}_1', \mathbf{p}_2', \mathbf{p}_3') \widetilde{\psi}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \\ &\quad \times \delta(\mathbf{p}_1' + \mathbf{p}_2' + \mathbf{p}_3' - \mathbf{p}') \delta(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 - \mathbf{p}) \\ &\quad \times \delta(\mathbf{p}_1 - \mathbf{p}_1') \delta(\mathbf{p}_2 - \mathbf{p}_2')\ d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 d\mathbf{p}_1' d\mathbf{p}_2' d\mathbf{p}_3' \end{aligned} \tag{3.47}$$

For future convenience, we introduce the new variables

$$\mathbf{r}_{12} \equiv \mathbf{r}_1 - \mathbf{r}_2;\ \ \mathbf{r}_{12,3} \equiv \mathbf{r}_{12} - \mathbf{r}_3;\ \ \mathbf{R}_{12} \equiv \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_{12}} \tag{3.48}$$

where $m_{12} \equiv \frac{m_1 m_2}{m_1+m_2}$ is the reduced mass of the light diquark system. Also, one introduces the momenta corresponding to these newly defined position-vectors

$$\mathbf{p}_{12} \equiv m_{12}\left(\frac{\mathbf{p}_1}{m_1} - \frac{\mathbf{p}_2}{m_2}\right); \quad \mathbf{P}_{12} \equiv \mathbf{p}_1 + \mathbf{p}_2; \tag{3.49}$$

and

$$\mathbf{k} \equiv \frac{m_3}{\widetilde{m}}\mathbf{P}_{12} - \frac{m_1 + m_2}{\widetilde{m}}\mathbf{p}_3 \tag{3.50}$$

where $\widetilde{m} \equiv m_1 + m_2 + m_3$.

In the rest frame of the b-baryon, one has

$$\mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = \mathbf{0} \tag{3.51}$$

therefore

$$\mathbf{P}_{12} = -\mathbf{p}_3 \tag{3.52}$$

From (3.50) and (3.52), one gets

$$\mathbf{k} = \mathbf{P}_{12} = -\mathbf{p}_3 \tag{3.53}$$

Now, if one expresses $\mathbf{p}_1$ and $\mathbf{p}_2$ in terms of $\mathbf{p}_{12}$ and $\mathbf{k}$ one obtains

$$\begin{aligned} \mathbf{p}_1 &= \mathbf{p}_{12} + \frac{m_1}{m_1 + m_2}\mathbf{k} && (3.54) \\ \mathbf{p}_2 &= -\mathbf{p}_{12} + \frac{m_2}{m_1 + m_2}\mathbf{k} && (3.55) \end{aligned}$$

Since q_1 and q_2 are nothing but spectator quarks in the processes, one has

$$m_1 = m_1', \; m_2 = m_2' \tag{3.56}$$

and

$$\mathbf{p}_1 = \mathbf{p}'_1, \ \mathbf{p}_2 = \mathbf{p}'_2 \tag{3.57}$$

thus

$$\mathbf{p}_{12} = \mathbf{p}'_{12}, \ \mathbf{P}_{12} = \mathbf{P}'_{12} \tag{3.58}$$

and

$$\mathbf{p}'_3 = \mathbf{p}' - \mathbf{P}_{12} \tag{3.59}$$

On combining (3.53) and (3.39), Eq. (3.59) becomes

$$\mathbf{p}'_3 = -\mathbf{q} - \mathbf{k}$$

Now, (3.53), (3.58), (3.59), and (3.50) (when applied to the daughter baryon quarks), yield

$$\begin{aligned} \mathbf{k}' &\equiv \frac{m'_3}{\tilde{m}'}\mathbf{P}_{12} - \frac{m'_1 + m'_2}{\tilde{m}'}\mathbf{p}'_3 &(3.60)\\ &= \mathbf{k} - \frac{m'_1 + m'_2}{\tilde{m}'}\mathbf{p}' &(3.61) \end{aligned}$$

which, by virtue of (3.56), transforms into

$$\mathbf{k}' = \mathbf{k} - \frac{m_1 + m_2}{\tilde{m}'}\mathbf{p}';$$

the second term of the right hand side is called *the mismatch factor*. It represents the recoil correction and arises since the rest frame of B_b baryon is not that of B baryon.

3.4.1 Normalization of the Baryon Wave-Functions

Normalization of the Initial Baryon Wave-Function

The wave-functions $\widetilde{\psi}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)$ must satisfy the normalization condition

$$\begin{aligned} 1 &= \langle \widetilde{\psi}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \mid \widetilde{\psi}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \rangle \\ &= N_i^2 \int \left| \widetilde{\psi}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) \right|^2 \delta(\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3)\, d\mathbf{p}_1 d\mathbf{p}_2 d\mathbf{p}_3 \end{aligned} \tag{3.62}$$

For convenience we will rewrite the normalization condition (3.62) in terms of new integration variables viz., $\mathbf{p}_{12}$ and $\mathbf{k}$. Eqs. (3.53), (3.54), and (3.55) give

$$\mathbf{p}_1 = \mathbf{p}_{12} + \frac{m_1}{m_1 + m_2}\mathbf{k} \tag{3.63}$$

$$\mathbf{p}_2 = -\mathbf{p}_{12} + \frac{m_2}{m_1 + m_2}\mathbf{k} \tag{3.64}$$

and hence, the normalization condition (3.62) becomes

$$1 = N_i^2 \int \left| \widetilde{\psi}(\mathbf{p}_{12}, \mathbf{k}) \right|^2 d\mathbf{p}_{12} d\mathbf{k} \tag{3.65}$$

Similarly, one has for the final baryon

$$\begin{aligned} 1 &= N_f^2 \int \left| \widetilde{\psi}(\mathbf{p}'_{12}, \mathbf{k}') \right|^2 d\mathbf{p}'_{12} d\mathbf{k}' \\ &= N_f^2 \int \left| \widetilde{\psi}(\mathbf{p}_{12}, \mathbf{k}') \right|^2 d\mathbf{p}_{12} d\mathbf{k}' \end{aligned} \tag{3.66}$$

where the replacement of $\mathbf{p}'_{12}$ with $\mathbf{p}_{12}$ is justified by (3.58).

To proceed further, we shall adopt a specific form for the wave functions. A Gaussian wave seems to be most appropriate. The reason is that for baryons containing a single heavy quark, the *bound state soliton picture* of Callan and Klebanov

[47] offers a convenient way to study the implications of the large N_c limit in baryon decays. In this picture, the heavy baryons containing a single heavy quark (termed *Λ_Q-type baryons*) are viewed as bound states of a nucleon of mass M_N and a meson of mass M_H: D (or D^*) meson for Λ_c-type baryons and B (or B^*) mesons for Λ_b-type baryons. The spatial wave functions of these Λ_Q-type baryons are governed by the harmonic oscillator potential

$$V(\mathrm{x}) = V_0 + \frac{1}{2}\kappa \mathrm{x}^2 \tag{3.67}$$

where [48]

$$\kappa = (440\ MeV)^3 \tag{3.68}$$

is the spring constant and V_0 the depth of the potential well. κ is related to the frequency of oscillation ω by

$$\omega^2 = \frac{\kappa}{\mu_Q}$$

μ_Q being the reduced mass

$$\mu_Q = \frac{M_N M_H}{M_N + M_H} \tag{3.69}$$

In addition, one has the following identity[2]

$$m - M_H \simeq M_N \simeq m - m_3.$$

[2] This equality has a simple physical origin. In the nucleon a light quark interacts with the mean color field created by the $N_c - 1$ other quarks. In the limit of large N_c, replacing one of these other light quarks by a heavy quark has a negligible effect on that mean color field. Thus for large N_c, the mass of the light quarks in a Λ_Q-type baryon (determined mainly by the strong interaction), is to a good approximation, equal to M_N [48].

Consequently, the spatial wave functions of the baryons are of the form

$$\tilde{\psi}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = N_i \exp\left[-\frac{1}{2}\frac{1}{\sqrt{\kappa\mu_Q}}\left(\mathbf{p}_1^2 + \mathbf{p}_2^2 + \mathbf{p}_3^2\right)\right] \tag{3.70}$$

which gives for the initial baryon B_b the wave function

$$\begin{aligned}\tilde{\psi}(\mathbf{p}_{12}, \mathbf{k}) &= N_i \exp[-\frac{1}{2\sqrt{\kappa\mu_Q}}(2\mathbf{p}_{12}^2 + 2\left(\frac{m_1 - m_2}{m_1 + m_2}\right)\mathbf{p}_{12}.\mathbf{k} \\ &\quad \left(1 - \frac{m_1^2 + m_2^2}{(m_1 + m_2)^2}\right)\mathbf{k}^2)]\end{aligned} \tag{3.71}$$

Thus, the normalization condition for B_b becomes

$$\begin{aligned}1 &= N_i^2 \int d\mathbf{p}_{12} d\mathbf{k} \exp[-\frac{1}{\sqrt{\kappa\mu_Q}}(2\mathbf{p}_{12}^2 + 2\left(\frac{m_1 - m_2}{m_1 + m_2}\right)\mathbf{p}_{12}.\mathbf{k} \\ &\quad + \left(1 + \frac{m_1^2 - m_2^2}{(m_1 - m_2)^2}\right)\mathbf{k}^2)]\end{aligned} \tag{3.72}$$

which after integration leads to

$$N_i = \left(\frac{3}{\pi^2\kappa\mu_Q}\right)^{3/4} \tag{3.73}$$

Normalization of the Final Baryon Wave-Function

Inserting (3.59) and (3.56) in (3.60) leads to

$$\mathbf{k}' = \mathbf{P}'_{12} - \frac{m_1 + m_2}{\tilde{m}'}\mathbf{p}' \tag{3.74}$$

thus

$$\mathbf{p}'_1 = \mathbf{k}' + \frac{m_1 + m_2}{\tilde{m}'}\mathbf{p}' - \mathbf{p}'_2 \tag{3.75}$$

By analogy to (3.49), we also have for the final baryon spectator quarks

$$\mathbf{p}'_2 = m_2\left(\frac{\mathbf{p}'_1}{m_1} - \frac{\mathbf{p}'_{12}}{m_{12}}\right)$$

which yields with (3.75)

$$\mathbf{p}'_1 = \frac{m_1}{m_1 + m_2}\mathbf{k}' + \frac{m_1}{\widetilde{m}'}\mathbf{p}' + \mathbf{p}_{12} \tag{3.76}$$

and

$$\mathbf{p}'_2 = \frac{m_2}{m_1 + m_2}\mathbf{k}' + \frac{m_2}{\widetilde{m}'}\mathbf{p}' - \mathbf{p}_{12} \tag{3.77}$$

in addition

$$\mathbf{p}'_1 + \mathbf{p}'_2 + \mathbf{p}'_3 = \mathbf{p}'$$

which, with (3.76) and (3.77), yields for $\mathbf{p}'_3$ the identity

$$\mathbf{p}'_3 = -\mathbf{k}' + \frac{m'_3}{\widetilde{m}'}\mathbf{p}' \tag{3.78}$$

Therefore, the wave function of the final baryon B reads

$$\begin{aligned}\widetilde{\psi}(\mathbf{p}_{12}, \mathbf{k}) \;=\;& N_f \exp\{-\frac{1}{2\sqrt{\kappa\mu'}}[2\mathbf{p}_{12}^2 + \left(1 + \frac{m_1^2 + m_2^2}{(m_1 + m_2)^2}\right)\mathbf{k}'^2 \\ &+ \frac{m_1^2 + m_2^2 - m_3'^2}{(\widetilde{m}')^2}\mathbf{p}'^2 + 2\left(\frac{m_1^2 + m_2^2 - m'_3(m_1 + m_2)}{\widetilde{m}'(m_1 + m_2)}\right)\mathbf{p}_{12}.\mathbf{k}' \\ &+ 2\left(\frac{m_1 - m_2}{m_1 - m_2}\right)\mathbf{p}_{12}.\mathbf{k}' + 2\left(\frac{m_1 - m_2}{\widetilde{m}'}\right)\mathbf{p}_{12}.\mathbf{p}']\}\end{aligned} \tag{3.79}$$

Under these conditions, the normalization condition (3.66) becomes

$$\begin{aligned}1 \;=\;& N_f^2 \int d\mathbf{p}_{12} d\mathbf{k}' \exp\{-\frac{1}{\sqrt{\kappa\mu'}}[2\mathbf{p}_{12}^2 + \left(1 + \frac{m_1^2 + m_2^2}{(m_1 + m_2)^2}\right)\mathbf{k}'^2 \\ &+ \frac{m_1^2 + m_2^2 - m_3'^2}{(\widetilde{m}')^2}\mathbf{p}'^2 + 2\left(\frac{m_1^2 + m_2^2 - m'_3(m_1 + m_2)}{\widetilde{m}'(m_1 + m_2)}\right)\mathbf{p}_{12}.\mathbf{k}' \\ &+ 2\left(\frac{m_1 - m_2}{m_1 + m_2}\right)\mathbf{p}_{12}.\mathbf{k}' + 2\left(\frac{m_1 - m_2}{\widetilde{m}'}\right)\mathbf{p}_{12}.\mathbf{p}']\}\end{aligned} \tag{3.80}$$

which gives after integration

$$N_f = \left(\frac{3}{\pi^2\kappa\mu_Q}\right)^{3/4} \exp\left[-\frac{1}{12\sqrt{\kappa\mu_Q}}\left(\frac{(m_1 - m_2)^2 - 6m_1m_2}{(m_1 - m_2)^2 + 3m_1m_2}\right)\right]$$

We are now in possession of all the ingredients necessary for the calculation of I. Indeed. (3.47) is easily shown to reduce to

$$\begin{aligned} I &= \int \widetilde{\psi}(\mathbf{p}_{12}, \mathbf{k})\, \widetilde{\psi}(\mathbf{p}'_{12}, \mathbf{k}') \\ &\quad \delta(\mathbf{p}_{12} - \mathbf{p}'_{12})\, \delta\left(\mathbf{k}' - \left(\mathbf{k} - \frac{m_1 + m_2}{\widetilde{m}'}\mathbf{p}'\right)\right) \\ &\quad d\mathbf{p}_{12}\, d\mathbf{p}'_{12}\, d\mathbf{k}\, d\mathbf{k}' \end{aligned} \tag{3.81}$$

After insertion of the explicit expressions of $\widetilde{\psi}(\mathbf{p}_{12}, \mathbf{k})$ and $\widetilde{\psi}(\mathbf{p}'_{12}, \mathbf{k})$ in (3.81) one integrates out to finally obtain

$$I = \left(\frac{2(\kappa\mu_Q)^{1/4}(\kappa\mu'_Q)^{1/4}}{(\kappa\mu_Q)^{1/2} + (\kappa\mu'_Q)^{1/2}}\right)^3 \exp\left[\frac{-3}{8\left((\kappa\mu_Q)^{1/2} + (\kappa\mu'_Q)^{1/2}\right)}\left(\frac{m_1 + m_2}{\widetilde{m}'}\mathbf{p}'\right)^2\right] \tag{3.82}$$

where a simple algebraic manipulation of the kinematic identities (3.39), and (3.11) applied to the case of meson X, shows that $|\mathbf{p}'|$ can be expressed in terms of the masses m, m' and m_X, as

$$|\mathbf{p}'| = \sqrt{\left(\frac{m^2 + m'^2 - m_X^2}{2m}\right)^2 - m'^2} \tag{3.83}$$

Chapter 4

Non-leptonic Decays of Bottom Baryons

In this part of our study, we apply the formalism introduced in Chapter 3 to the investigation of specific non-leptonic decays of some bottom baryons. We will focus our attention on weak decays of the types

$$\begin{aligned} \mathbf{3}_b^* &\rightarrow \mathbf{3}_c^* + V,\ P \text{ or } A \\ \mathbf{6}_b\left(J^P = \frac{1}{2}^-\right) &\rightarrow \mathbf{6}_c\left(J^P = \frac{1}{2}^+\right) + V\ ,\ P \text{ or } A \end{aligned}$$

where V, P, and A are respectively vector, pseudoscalar and axial vector mesons. For definiteness, we consider processes for which $\Delta b = 1$, $\Delta c = -1$, $\Delta s = 0$, and concentrate on those for which either only factorization contributes or baryon pole contributions are negligible. The ultimate goal will be the evaluation of the decay widths of the selected decays. We start by setting up the formulae relevant for each category of processes in section 4.1. In section 4.2 we perform the numerical calculations, and in section 4.3 we discuss our results.

4.1 Evaluation of the Decay Widths

The effective Hamiltonian for $\Delta b = 1$, $\Delta c = -1$, $\Delta s = 0$ transitions is given in the *OPE* scheme by [43]

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^{*} \left(C_1 O_1 + C_2 O_2\right) \tag{4.1}$$

where

$$C_1 \equiv C_1\left(\mu\right) = 1.117$$

and

$$C_2 \equiv C_2\left(\mu\right) = -0.257$$

are the Wilson coefficients evaluated at the renormalization scale $\mu = 2.5\ GeV$ in the *next-to-leading logarithmic (NLL)* approximation [49]. These are not very different from those at $\mu = 5\ GeV \simeq m_b$ in the *leading logarithmic approximation (LLA)* [50]: $C_1\left(m_b\right) = 1.11$ and $C_2\left(m_b\right) = -0.26$.

The current-current effective operators O_i are given by

$$\begin{aligned} O_1 &\equiv (\overline{c}^{\alpha} b_{\alpha})_{V-A} (\overline{d}^{\beta} u_{\beta})_{V-A} \equiv \left[\overline{c}^{\alpha} \gamma_{\mu} (1+\gamma_5) b_{\alpha}\right] \left[\overline{d}^{\beta} \gamma_{\mu} (1+\gamma_5) u_{\beta}\right] \\ O_2 &\equiv (\overline{c}^{\alpha} b_{\beta})_{V-A} (\overline{d}^{\beta} u_{\alpha})_{V-A} \equiv \left[\overline{c}^{\alpha} \gamma_{\mu} (1+\gamma_5) b_{\beta}\right] \left[\overline{d}^{\beta} \gamma_{\mu} (1+\gamma_5) u_{\alpha}\right] \end{aligned} \tag{4.2}$$

here α and β are *SU(3)* color indices. In the rest frame of a polarized B_b parent-baryon with density matrix [44]

$$\rho_{B_b} = \frac{1}{2}\left(1 + i\gamma_5 \gamma.\mathrm{n}\right) \tag{4.3}$$

the decay rate into an unpolarized B daughter-baryon and an unpolarized X meson is given by

$$\begin{aligned} d\Gamma &= \sum_{\substack{spin\ projections\\ of\ B_b}} \sum_{\substack{spin\ projections\\ of\ X}} \int d\mathbf{p} \int d\mathbf{p}_X \, (2\pi)^4 \, \delta^4(p-(p'+p_X)) \left|\widetilde{T}_{fi}\right|^2 \\ &\equiv \int d\mathbf{p} \int d\mathbf{q} \, (2\pi)^4 \, \delta^4(p-(p'+p_X)) \sum_f \left|\widetilde{T}_{fi}\right|^2 \end{aligned} \tag{4.4}$$

In the factorization approximation, the transition matrix elements $\widetilde{T}_{fi}$ are proportional to $\langle 0 \left| J'_\mu \right| X(q) \rangle$ and $\langle B(p') \left| J_\mu \right| B_b(p) \rangle$ (3.9). In order to calculate $\left|\widetilde{T}_{fi}\right|^2$ we define the tensors

$$K_{\mu\nu} \equiv \sum_f \langle B(p') \left| J_\mu \right| B_b(p) \rangle \langle B(p') \left| J_\nu^\dagger \right| B_b(p) \rangle \tag{4.5}$$

and

$$M_{\mu\nu} \equiv \int d\mathbf{p}_X \sum_f (2\pi)^3 \, \delta^4(p_X - q) \langle 0 \left| J'_\mu \right| X(q) \rangle \langle X(q) \left| J_\nu'^\dagger \right| 0 \rangle \tag{4.6}$$

For the baryonic tensor $K_{\mu\nu}$ one can write

$$\begin{aligned} K_{\mu\nu} &= -Tr\{\left[\left(\frac{m' - i\gamma.p'}{2m'}\right)(A + B\gamma_5)\gamma_\mu\right] \\ &\quad \times \left[\left(\left(\frac{m - i\gamma.p}{2m}\right)\right)\frac{1}{2}(1 + i\gamma_5\boldsymbol{\gamma}.\mathbf{n})(A^* + B^*\gamma_5)\gamma_\nu\right]\} \\ &= \frac{|A|^2}{2}(L_{\mu\nu} + N_{\mu\nu}) \end{aligned}$$

where

$$L_{\mu\nu} \equiv -\frac{1}{4mm'} Tr\left[(m' - i\gamma.p')(1 + a\gamma_5)\gamma_\mu (m - i\gamma.p)(1 + a\gamma_5)\gamma_\nu\right] \tag{4.7}$$

is the only term which would have contributed if the parent baryon were not polarized, while

$$N_{\mu\nu} \equiv -\frac{1}{4mm'} Tr\left[(m' - i\gamma.p')(1 + a\gamma_5)\gamma_\mu (m - i\gamma.p)\, i\gamma_5\boldsymbol{\gamma}.\mathbf{n}\,(1 + a\gamma_5)\gamma_\nu\right] \tag{4.8}$$

is the term which arises because of the polarization. By standard trace techniques (cf. Appendix B), one can show that

$$\begin{aligned} L_{\mu\nu} &= \frac{1}{mm'}[\left(1+a^2\right)\left(p_\mu p'_\nu + p_\nu p'_\mu - \delta_{\mu\nu} p.p'\right) \\ &\quad - mm'\left(1-a^2\right) + 2a\varepsilon_{\mu\nu\tau\theta} p_\tau p_\theta] \end{aligned} \tag{4.9}$$

$$\begin{aligned} N_{\mu\nu} &= -\frac{1}{mm'}[2a\left(n_\mu p'_\nu + n_\nu p'_\mu - \delta_{\mu\nu} n.p'\right) \\ &\quad - \left(1+a^2\right)\varepsilon_{\mu\nu\tau\theta} p'_\tau n_\theta] \end{aligned} \tag{4.10}$$

Thus

$$\begin{aligned} K_{\mu\nu} &= \frac{|A|^2}{2mm'}\{\left(1+a^2\right)[\left(p_\mu p'_\nu + p_\nu p'_\mu - \delta_{\mu\nu} p.p'\right) + \varepsilon_{\mu\nu\tau\theta} p'_\tau n_\theta] \\ &\quad -2a[\left(n_\mu p'_\nu + n_\nu p'_\mu - \delta_{\mu\nu} n.p'\right) - \varepsilon_{\mu\nu\tau\theta} p_\tau p_\theta] \\ &\quad - mm'\left(1-a^2\right)\} \end{aligned} \tag{4.11}$$

where

$$a \equiv \frac{1}{|A|^2}\,\mathrm{Re}\left(A^* B\right) = \frac{1}{|A|^2}\,\mathrm{Re}\left(AB^*\right)$$

By difining a in this way, we are neglecting the terms which would yield the transversal asymmetry of the decays, keeping only the contributions of the unpolarized decay and those due to the longitudinal asymmetry (which is to be defined later).

On using the appropriate expression for $\langle 0 | J'_\mu | X(q) \rangle$ (cf. (3.13)) and the fact that for vector mesons one has the identity (cf. (2.14))

$$\sum_{\lambda=0,\pm1} \epsilon_\mu^{(\lambda)} . \epsilon_\nu^{(\lambda)} = \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \tag{4.12}$$

$\epsilon^{(\lambda)}$ being the meson polarization 4-vector, one can show that

$$M_{\mu\nu} = \left(-q^2\delta_{\mu\nu} + q_\mu q_\nu\right) \varrho_X(q^2) + q_\mu q_\nu \zeta_X(q^2) \tag{4.13}$$

where $\varrho_X(q^2)$ and $\zeta_X(q^2)$ are spectral functions giving respectively the transverse and longitudinal components of $M_{\mu\nu}$. For a vector, only the transverse component contributes ; for a pseudoscalar only the longitudinal component contributes ; for an axial vector both components contribute to the decay rate.

Hence, the decay width is given in covariant form by

$$\begin{aligned} \Gamma\left(B_b \to BX\right) &= \frac{G'^2}{2\left(2\pi\right)^2} \int \frac{m'}{E'} d\mathbf{p}' K_{\mu\nu} M_{\mu\nu} \\ &= \frac{G'^2 m'}{4\pi m} \int ds \left|\mathbf{p}'\right| K_{\mu\nu} M_{\mu\nu} \end{aligned}$$

which ultimately yields[1]

$$\Gamma\left(B_b \to BX\right) = \frac{G'^2}{8\pi m^2} \int ds \ \left|\mathbf{p}'\right| \left(\varrho_X \Gamma_{\varrho_X} + \zeta_X \Gamma_{\zeta_X}\right) \tag{4.14}$$

where

$$\varrho_X \equiv \begin{cases} \varrho_V(s) \equiv F_V^2 \delta(s - m_V^2), \text{ for } X\left(J^P\right) = V\left(1^-\right) \\ \varrho_A(s) \equiv F_A^2 \delta(s - m_A^2), \text{ for } X\left(J^P\right) = A\left(1^+\right) \\ 0, \text{ for } X\left(J^P\right) = P\left(0^-\right) \end{cases} \tag{4.15}$$

$$\zeta_X \equiv \begin{cases} 0, \text{ for } X\left(J^P\right) = V\left(1^-\right) \\ \zeta_A(s) \equiv F_{P'}^2 \delta(s - m_{P'}^2), \text{ for } X\left(J^P\right) = A\left(1^+\right) \\ \zeta_P(s) \equiv F_P^2 \delta(s - m_P^2), \text{ for } X\left(J^P\right) = P\left(0^-\right) \end{cases} \tag{4.16}$$

[1] The use of this covariant form instead of its nonrelativistic limit found in the literature (as in [51] and [52] for expamle) is dictated by its simplicity and generality; in (4.14) a single formula gives the decay rate regardless of the type of meson accompanying the decayed baryon B_c, whereas in the literature different expressions for Γ exist according as the meson is pseudoscalar, or vector.

P' being the pseudoscalar meson which shares the same quark structure with the axial vector meson under consideration; and

$$\begin{aligned}
\Gamma_{\varrho X} \equiv \Gamma_{\varrho X}(s) = & \{Q(g_V^2 + g_A^2) - 3smm'(g_V^2 - g_A^2) \\
& + 3s[(m+m')((m-m')^2 - s)g_V f_V \\
& + (m-m')((m+m')^2 - s)g_A f_A] \\
& + s[Q''(f_V^2 + h_A^2) - 3smm'(f_V^2 - h_A^2)] \\
& - 2m\,|\mathbf{p}'|\,\mathbf{n}.\mathbf{s}_{B_b}[(m^2 - m'^2 - 2s)g_A g_V \\
& + s((m - 3m')g_V h_A - (m+3m')g_A f_V) \\
& + s(s - m^2 - 5m'^2) f_V h_A]\}
\end{aligned} \tag{4.17}$$

$$\begin{aligned}
\Gamma_{\zeta X} \equiv \Gamma_{\zeta X}(s) = & \{Q'(g_V^2 + g_A^2) - s[(m-m')((m+m')^2 - s)g_V f_V \\
& + (m+m')((m-m')^2 - s)g_A f_A] \\
& + \frac{1}{2}s^2[((m+m')^2 - s)h_V^2 + ((m-m')^2 - s)f_A^2] \\
& - 2m\,|\mathbf{p}'|\,\mathbf{n}.\mathbf{s}_{B_b}[(m^2 - m'^2)g_A g_V + s^2 h_V f_A \\
& - s((m+m')g_A h_V + (m-m')f_A g_V)]\}
\end{aligned} \tag{4.18}$$

where $\mathbf{n} \equiv \frac{\mathbf{p}'}{|\mathbf{p}'|}$ and $\mathbf{s}_{B_b}$ is the polarization direction of the initial baryon. In the above expressions it is understood that the form factors are functions of $s = m_X^2$. In addition

$$|\mathbf{p}'| \equiv |\mathbf{p}'(s)| = \frac{1}{2m}\left[\left(m^2 + m'^2 - s^2\right)^2 - 4m^2 m'^2\right]^{1/2} \tag{4.19}$$

$$Q \equiv Q(s) = \frac{1}{2}\left[(m^2 - m'^2)^2 + s(m^2 + m'^2) - 2s^2\right] \tag{4.20}$$

$$Q' \equiv Q'(s) = \frac{1}{2}\left[(m^2 - m'^2)^2 - s(m^2 + m'^2)\right] \tag{4.21}$$

$$Q'' \equiv Q''(s) = \frac{1}{2}\left[2(m^2 - m'^2)^2 - s(m^2 + m'^2) - s^2\right] \tag{4.22}$$

One notices that $\Gamma_{\varrho}(s)$ and $\Gamma_{\zeta}(s)$ (i.e., Γ) split naturally into two terms. The first term measures the decay rate of an unpolarized parent baryon, while the second term (represented by the coefficient of $\mathbf{n}.\mathbf{s}_{B_b}$) measures the spin up-spin down and longitudinal asymmetry of the decay, i.e., the asymmetric emission of the daughter baryon with respect to the direction of the parent spin. In what follows this asymmetry will be referred to as the parameter

$$\alpha(B_b \to BX) = \frac{\textit{coefficient of } \mathbf{n}.\mathbf{s}_{B_b}}{\textit{unpolarized } B_b \textit{decay width}} \tag{4.23}$$

4.1.1 $\mathbf{3}_b^* \to \mathbf{3}_c^*$ Transitions

We saw in chapter 3 that there exist a spin-flavor contribution to the transition matrix elements represented by the factors ξ_V and $\xi_A = \xi_V \langle spin\,|(\sigma_3)_b|\,spin\rangle$ (cf. (3.31) and (3.32)). For $\mathbf{3}_b^* \to \mathbf{3}_c^*$ transitions these coefficients become

$$\begin{aligned}
\xi_V &= \left\langle spin,\ flavor'\left|a_c^\dagger a_b\right|spin,\ flavor\right\rangle \\
&= \left\langle MA_{ij}(B_c)\left|a_c^\dagger a_b\right| MA_{ij}(B_b)\right\rangle \\
&= \left\langle \frac{1}{\sqrt{2}}(q_i q_j - q_j q_i)c\chi_{MA}\left|a_c^\dagger a_b\right| \frac{1}{\sqrt{2}}(q_i q_j - q_j q_i)b\chi_{MA}\right\rangle \\
&= 1
\end{aligned}$$

and

$$\xi_A = \left\langle spin,\ flavor'\left|a_c^\dagger a_b(\sigma_3)_b\right|spin,\ flavor\right\rangle$$

$$= \xi_V \langle \chi_{MA} | (\sigma_3)_b | \chi_{MA} \rangle$$

$$= \xi_V$$

therefore

$$\xi_V = 1 = \xi_A \tag{4.24}$$

Consequently, (3.40), (3.41), and (3.42) yield as noted in chapter 3 (cf. (3.45))

$$\begin{aligned} g_A(s) &= g_V(s) \equiv f_1 \\ h_V(s) &= f_V(s) = h_A(s) = -f_A(s) \equiv \frac{1}{m} f_2. \end{aligned} \tag{4.25}$$

where

$$\begin{aligned} f_1 &\equiv Ia(E', E_3') \\ f_2 &\equiv Ib(E', E_3') \end{aligned} \tag{4.26}$$

$3_b^* \to 3_c^* V$ Decays

In this category, we are interested in the decays

$$\begin{aligned} \Lambda_b^0 &\to \Lambda_c^+ \rho^- \\ \Xi_b^0 &\to \Xi_c^+ \rho^- \end{aligned}$$

for which

$$\begin{aligned} G'\left(\Lambda_b^0 \to \Lambda_c^+ \rho^-\right) &= G_F V_{cb} V_{ud}^* \left(C_1 + \frac{C_2}{N_c}\right) \\ &= G_F V_{cb} V_{ud}^* \left(C_1 + \xi C_2\right) \\ &= G'\left(\Xi_b^0 \to \Xi_c^+ \rho^-\right) \end{aligned} \tag{4.27}$$

Moreover, for $\mathbf{3}_b^* \rightarrow \mathbf{3}_c^* V$ transitions, the decay width (4.14) reduces by virtue of (4.15) and (4.16) to

$$\Gamma\left(\mathbf{3}_b^* \rightarrow \mathbf{3}_c^* V\right) = \frac{G'^2}{4\pi m^2}\left|\mathrm{p}'\left(m_V^2\right)\right| F_V^2 Q(m_V^2) f_1^2(m_V^2) \Re_V^{(1)}(m_V^2) \tag{4.28}$$

while the asymmetry

$$\alpha\left(\mathbf{3}_b^* \rightarrow \mathbf{3}_c^* V\right) = -\frac{\Re_V^{(2)}(m_V^2)}{\Re_V^{(1)}(m_V^2)} \frac{m\left|\mathrm{p}'\left(m_V^2\right)\right|\left(m^2 - m'^2 - 2m_V^2\right)}{Q(m_V^2)} \tag{4.29}$$

where

$$\Re_V^{(1)}(m_V^2) \equiv 1 - 3\left(\frac{f_2}{f_1}\frac{m_V}{m}\right)\frac{m' m_V\left(m^2 - m'^2 + m_V^2\right)}{Q(m_V^2)} + \left(\frac{f_2}{f_1}\frac{m_V}{m}\right)^2 \frac{Q''(m_V^2)}{Q(m_V^2)} \tag{4.30}$$

and

$$\Re_V^{(2)}(m_V^2) \equiv 1 - 6\left(\frac{f_2}{f_1}\frac{m_V}{m}\right)\frac{m' m_V}{m^2 - m'^2 - 2m_V^2} - \left(\frac{f_2}{f_1}\frac{m_V}{m}\right)^2 \frac{m^2 + 5m'^2 - m_V^2}{m^2 - m'^2 - 2m_V^2} \tag{4.31}$$

$\mathbf{3}_b^* \rightarrow \mathbf{3}_c^* P$ Decays

In this category, we are interested in the processes

$$\Xi_b^0 \rightarrow \Xi_c^+ \pi^-$$

$$\Xi_b^- \rightarrow \Xi_c^0 \pi^-$$

for which

$$\begin{aligned} G'\left(\Xi_b^0 \rightarrow \Xi_c^+ \pi^-\right) &= G_F V_{cb} V_{ud}^* \left(C_1 + \frac{C_2}{N_c}\right) \\ &= G_F V_{cb} V_{ud}^* \left(C_1 + \xi C_2\right) && (4.32) \\ &= G'\left(\Xi_b^- \rightarrow \Xi_c^0 \pi^-\right) && (4.33) \end{aligned}$$

For a pseudoscalar meson in the final state one has from (4.14), (4.15) and (4.16)

$$\Gamma\left(\mathbf{3}_b^* \to \mathbf{3}_c^* P\right) = \frac{G'^2}{4\pi m^2} \left|\mathbf{p}'\left(m_P^2\right)\right| F_P^2 Q'(m_P^2) f_1^2(m_P^2) \Re_P^{(1)}(m_P^2) \tag{4.34}$$

$$\alpha\left(\mathbf{3}_b^* \to \mathbf{3}_c^* P\right) = -\frac{\Re_P^{(2)}(m_P^2)}{\Re_P^{(1)}(m_P^2)} \frac{m\left|\mathbf{p}'\left(m_P^2\right)\right|\left(m^2 - m'^2\right)}{Q'(m_P^2)} \tag{4.35}$$

where

$$\begin{aligned}\Re_P^{(1)}(m_P^2) \equiv\ & 1 - \left(\frac{f_2}{f_1}\frac{m_P}{m}\right) \frac{m' m_P\left(m^2 - m'^2 + m_P^2\right)}{Q'(m_P^2)} \\ & + \left(\frac{f_2}{f_1}\frac{m_P}{m}\right)^2 \frac{m_P^2\left(m^2 + m'^2 - m_P^2\right)}{2Q'(m_P^2)}\end{aligned} \tag{4.36}$$

and

$$\Re_P^{(2)}(m_P^2) \equiv 1 - 2\left(\frac{f_2}{f_1}\frac{m_P}{m}\right)\frac{m_P m'}{m^2 - m'^2} - \left(\frac{f_2}{f_1}\frac{m_P}{m}\right)^2 \frac{m_P^2}{m^2 - m'^2} \tag{4.37}$$

$\mathbf{3}_b^* \to \mathbf{3}_c^* A$ Decays

In this category, we are interested in the processes

$$\Lambda_b^0 \to \Lambda_c^+ a_1^-$$

$$\Xi_b^0 \to \Xi_c^+ a_1^-$$

for which

$$\begin{aligned}G'\left(\Lambda_b^0 \to \Lambda_c^+ a_1^-\right) &= G_F V_{cb} V_{ud}^* \left(C_1 + \frac{C_2}{N_c}\right) \\ &= G_F V_{cb} V_{ud}^* \left(C_1 + \xi C_2\right) \qquad (4.38) \\ &= G'\left(\Xi_b^0 \to \Xi_c^+ a_1^-\right) \qquad (4.39)\end{aligned}$$

In addition, for $\mathbf{3}_b^* \to \mathbf{3}_c^* A$ one obtains from Eqs. (4.14), (4.15) and (4.16)

$$\Gamma\left(\mathbf{3}_b^* \to \mathbf{3}_c^* A\right) = \frac{G'^2}{4\pi m^2} [\left|\mathbf{p}'\left(m_A^2\right)\right| F_A^2 Q(m_A^2) f_1^2(m_A^2) \Re_V^{(1)}(m_A^2) \tag{4.40}$$

$$+\left|\mathbf{p}'\left(m_P^2\right)\right| F_{P'}^2 Q'(m_{P'}^2) f_1^2(m_{P'}^2)\Re_P^{(1)}(m_{P'}^2)] \tag{4.41}$$

$$\alpha\left(3_b^* \to 3_c^* A\right) = -2\frac{\Re_V^{(2)}(m_A^2)}{\Re_V^{(1)}(m_A^2)}\frac{\mathcal{F}_A^{(2)}\left(m_A^2, m_{P'}^2\right)}{\mathcal{F}_A^{(1)}\left(m_A^2, m_{P'}^2\right)}\frac{m\left|\mathbf{p}'\left(m_A^2\right)\right|\left(m^2 - m'^2 - 2m_A^2\right)}{Q(m_A^2)} \tag{4.42}$$

where

$$\mathcal{F}_A^{(1)}\left(m_A^2, m_{P'}^2\right) \equiv 1 + \frac{F_{P'}^2}{F_A^2}\frac{\Re_P^{(1)}(m_{P'}^2)}{\Re_V^{(1)}(m_A^2)}\frac{f_1^2(m_{P'}^2)}{f_1^2(m_A^2)}\frac{\left|\mathbf{p}'\left(m_{P'}^2\right)\right|}{\left|\mathbf{p}'\left(m_A^2\right)\right|}\frac{Q'(m_{P'}^2)}{Q(m_A^2)} \tag{4.43}$$

and

$$\mathcal{F}_A^{(2)}\left(m_A^2, m_{P'}^2\right) \equiv 1 + \frac{F_{P'}^2}{F_A^2}\frac{\Re_P^{(2)}(m_{P'}^2)}{\Re_V^{(2)}(m_A^2)}\frac{f_1^2(m_{P'}^2)}{f_1^2(m_A^2)}\frac{\left|\mathbf{p}'\left(m_{P'}^2\right)\right|^2}{\left|\mathbf{p}'\left(m_A^2\right)\right|^2}\frac{m^2 - m'^2}{m^2 - m'^2 - 2m_A^2} \tag{4.44}$$

and $\Re_V^{(1)}(m_A^2)$, $\Re_V^{(2)}(m_A^2)$, $\Re_P^{(1)}(m_{P'}^2)$, and $\Re_P^{(2)}(m_{P'}^2)$ are the same as in (4.30), (4.31), (4.36), and (4.37) respectively.

4.1.2 $6_b \to 6_c$ Transitions

For $6_b\left(\frac{1}{2}^+\right) \to 6_c\left(\frac{1}{2}^+\right)$ transitions one obtains

$$\begin{aligned}
\xi_V &= \left\langle spin,\ flavor'\left|a_c^\dagger a_b\right| spin,\ flavor\right\rangle \\
&= \left\langle MS_{ij}\left(B_c\right)\left|a_c^\dagger a_b\right| MS_{ij}\left(B_b\right)\right\rangle \\
&= \left\langle \frac{1}{\sqrt{2}}(q_i q_j + q_j q_i)c\chi_{MS}\left|a_c^\dagger a_b\right| \frac{1}{\sqrt{2}}(q_i q_j + q_j q_i)b\chi_{MS}\right\rangle \\
&= 1
\end{aligned}$$

but

$$\begin{aligned}
\xi_A &= \xi_V\left\langle spin\left|(\sigma_3)_b\right| spin\right\rangle \\
&= \xi_V\left\langle \chi_{MS}\left|(\sigma_3)_b\right| \chi_{MS}\right\rangle \\
&= -\frac{1}{3}\xi_V
\end{aligned}$$

hence

$$\xi_V = 1 = -3\xi_A \tag{4.45}$$

and one obtains as a consequence of (4.45), (3.40), (3.41), and (3.42)

$$\begin{aligned} g_V(s) &= -3g_A(s) \equiv f_1 \\ h_V(s) &= f_V(s) = 3f_A(s) = -3h_A(s) \equiv \frac{1}{m} f_2 \end{aligned} \tag{4.46}$$

where f_1 and f_2 are the functions defined in Eqs. (4.26).

$\mathbf{6}_b\left(\frac{1}{2}^+\right) \rightarrow \mathbf{6}_c\left(\frac{1}{2}^+\right) V$ Decays

In this category, we study the process

$$\Omega_b^- \rightarrow \Omega_c^0 \rho^-$$

for which

$$\begin{aligned} G'\left(\Omega_b^- \rightarrow \Omega_c^0 \rho^-\right) &= G_F V_{cb} V_{ud}^* \left(C_1 + \frac{C_2}{N_c}\right) \\ &= G_F V_{cb} V_{ud}^* \left(C_1 + \xi C_2\right) \end{aligned} \tag{4.47}$$

On the other hand for $\mathbf{6}_b\left(\frac{1}{2}^+\right) \rightarrow \mathbf{6}_c\left(\frac{1}{2}^+\right) V$ one has

$$\Gamma\left(\mathbf{6}_b \rightarrow \mathbf{6}_c V\right) = \frac{G'^2}{36\pi m^2} \left|\mathrm{p}'\left(m_V^2\right)\right| \left[5Q(m_V^2) - 12m_V^2 m m'\right] F_V^2 f_1^2(m_V^2) \Re_V'^{(1)}(m_V^2) \tag{4.48}$$

while the asymmetry

$$\alpha\left(\mathbf{6}_b \rightarrow \mathbf{6}_c V\right) = 3\frac{\Re_V^{(2)}(m_V^2)}{\Re_V'^{(1)}(m_V^2)} \frac{m\left|\mathrm{p}'\left(m_V^2\right)\right|\left(m^2 - m'^2 - 2m_V^2\right)}{5Q(m_V^2) - 12m_V^2 m m'} \tag{4.49}$$

where $\Re_V^{(2)}(m_V^2)$ is the same as in (4.31) and

$$\begin{aligned}\Re_V^{\prime(1)}(m_V^2) &= 1+3\left(\frac{f_2}{f_1}\frac{m_V}{m}\right)\frac{m_V\left[4m\left(m^2-m'^2-m_V^2\right)-5m'\left(m^2-m'^2+m_V^2\right)\right]}{5Q(m_V^2)-12m_V^2mm'} \\ &+\left(\frac{f_2}{f_1}\frac{m_V}{m}\right)^2\frac{5Q''(m_V^2)-12m_V^2mm'}{5Q(m_V^2)-12m_V^2mm'} \qquad (4.50)\end{aligned}$$

$\mathbf{6}_b\left(\frac{1}{2}^+\right)\to\mathbf{6}_c\left(\frac{1}{2}^+\right)P$ Decays

Here, we investigate the process

$$\Omega_b^-\to\Omega_c^0\pi^-$$

for which

$$\begin{aligned}G'\left(\Omega_b^-\to\Omega_c^0\pi^-\right) &= G_FV_{cb}V_{ud}^*\left(C_1+\frac{C_2}{N_c}\right) \\ &= G_FV_{cb}V_{ud}^*\left(C_1+\xi C_2\right) \qquad (4.51)\end{aligned}$$

while the decay width and asymmetry of $\mathbf{6}_b\left(\frac{1}{2}^+\right)\to\mathbf{6}_c\left(\frac{1}{2}^+\right)P$ transitions are given by

$$\Gamma\left(\mathbf{6}_b\to\mathbf{6}_cP\right)=\frac{5G'^2}{36\pi m^2}\left|\mathbf{p}'\left(m_P^2\right)\right|F_P^2Q'(m_P^2)f_1^2(m_P^2)\Re_P^{\prime(1)}(m_P^2) \qquad (4.52)$$

$$\alpha\left(\mathbf{6}_b\to\mathbf{6}_cP\right)=\frac{3}{5}\frac{\Re_P^{(2)}(m_P^2)}{\Re_P^{\prime(1)}(m_P^2)}\frac{m\left|\mathbf{p}'\left(m_P^2\right)\right|\left(m^2-m'^2\right)}{Q'(m_P^2)} \qquad (4.53)$$

where $\Re_P^{(2)}(m_P^2)$ is the same as in (4.37) and

$$\begin{aligned}\Re_P^{\prime(1)}(m_P^2) &= 1-\frac{1}{5}\left(\frac{f_2}{f_1}\frac{m_P}{m}\right)\frac{m_P\left[4m\left(m^2-m'^2-m_P^2\right)+5m'\left(m^2-m'^2+m_P^2\right)\right]}{Q'(m_P^2)} \\ &+\frac{1}{10}\left(\frac{f_2}{f_1}\frac{m_P}{m}\right)^2\frac{\left[5\left(m^2+m'^2-m_P^2\right)+8mm'\right]}{Q'(m_P^2)} \qquad (4.54)\end{aligned}$$

$\mathbf{6}_b\left(\frac{1}{2}^+\right) \rightarrow \mathbf{6}_c\left(\frac{1}{2}^+\right) A$ Decays

Finally, we want to investigate the process

$$\Omega_b^- \rightarrow \Omega_c^0 a_1^-$$

for which

$$\begin{aligned} G'\left(\Omega_b^- \rightarrow \Omega_c^0 a_1^-\right) &= G_F V_{cb} V_{ud}^* \left(C_1 + \frac{C_2}{N_c}\right) \\ &= G_F V_{cb} V_{ud}^* \left(C_1 + \xi C_2\right) \end{aligned} \tag{4.55}$$

But for $\mathbf{6}_b\left(\frac{1}{2}^+\right) \rightarrow \mathbf{6}_c\left(\frac{1}{2}^+\right) A$ one gets

$$\begin{aligned} \Gamma\left(\mathbf{6}_b \rightarrow \mathbf{6}_c A\right) &= \frac{G'^2}{36\pi m^2}\{\left|\mathrm{p}'\left(m_A^2\right)\right|\left[5Q(m_A^2) - 12m_A^2 m m'\right] F_A^2 f_1^2(m_A^2) \Re_V'^{(1)}(m_A^2) \\ &\quad +5\left|\mathrm{p}'\left(m_{P'}^2\right)\right| F_{P'}^2 Q'(m_{P'}^2) f_1^2(m_{P'}^2) \Re_P'^{(1)}(m_{P'}^2)\} \end{aligned} \tag{4.56}$$

$$\alpha\left(3_b^* \rightarrow 3_c^* A\right) = \frac{2}{3}\frac{\Re_V^{(2)}(m_A^2)}{\Re_V'^{(1)}(m_A^2)}\frac{\mathcal{F}_A^{(2)}\left(m_A^2, m_{P'}^2\right)}{\mathcal{F}_A'^{(1)}\left(m_A^2, m_{P'}^2\right)}\frac{m\left|\mathrm{p}'\left(m_A^2\right)\right|\left(m^2 - m'^2 - 2m_A^2\right)}{5Q(m_A^2) - 12m_A^2 m m'} \tag{4.57}$$

where the function $\mathcal{F}_A^{(2)}\left(m_A^2, m_{P'}^2\right)$ is same as in (4.44) whereas

$$\mathcal{F}_A'^{(1)}\left(m_A^2, m_{P'}^2\right) \equiv 1 + 5\frac{F_{P'}^2}{F_A^2}\frac{\Re_P'^{(1)}(m_{P'}^2)}{\Re_V'^{(1)}(m_A^2)}\frac{f_1^2(m_{P'}^2)}{f_1^2(m_A^2)}\frac{\left|\mathrm{p}'\left(m_{P'}^2\right)\right|}{\left|\mathrm{p}'\left(m_A^2\right)\right|}\frac{Q'(m_{P'}^2)}{Q(m_A^2)} \tag{4.58}$$

and $\Re_V^{(1)}(m_A^2)$, $\Re_V^{(2)}(m_A^2)$, $\Re_P^{(1)}(m_{P'}^2)$, $\Re_P^{(2)}(m_{P'}^2)$, $\Re_V'^{(1)}(m_A^2)$ and $\Re_P'^{(1)}(m_{P'}^2)$ are the same as in (4.30),(4.31), (4.36), (4.37), (4.50), and (4.54) respectively.

4.2 Applications

The masses of the constituent u, d, s, c, and b quarks are given in Table 1.1, while the masses of the ρ^-, π^-, and a_1^- mesons and their decay constants (in the normalization

$F_\pi = 0.132\ GeV$) are given in Table 4.1; The bottom and charm baryon masses relevant to this part of the study are listed in Table 4.2. The results of our calculation are summarized in Table 4.3. Column 7 gives the decay width in the large N_c limit (i.e., at $\xi = 0$). Notice that only columns 5, 6 and 7 depend on the overlap I.

4.3 Discussion

The obtained decay widths are not very sensitive to ξ for $0.5 \geqslant \xi \geqslant 0$, as shown in Figure 4.1, they drop by a little more than 20 %. This range is suggested by the comparaison of the theoretical and experimental branching ratios of the nonleptonic decay $\Lambda_b^0 \to \Lambda^0 + J/\Psi$ [53], and the analysis of the experimental results on $B \to h_1 + h_2$ where h_1 and h_2 are two hadrons [49]. The corrections due to the form factors, which scale as $1/m$, were dumped into the $\Re$ and $\mathcal{F}$ functions. The prediction for $\alpha-$asymmetry is independent of the overlap integral and provides a test for the predictions (4.25) and (4.46) through the presence of f_2/f_1 in $\Re$ and $\mathcal{F}$ functions. The f_2/f_1 corrections are negligible for all the decays; they contribute by less than 3 % to the decay rates and $\alpha-$asymmetries. However, a remarkable result has been obtained for the contribution of these corrections to the $\alpha-$asymmetry of the $\mathbf{6}_b \to \mathbf{6}_c$ transition $\Omega_b^- \to \Omega_c^0 a_1^-$. We have found that the f_2/f_1 corrections have a contribution of 34 % to the up-down asymmetry of the decay. This would, in fact, constitute a test for heavy quark effective theory (*HQET*) in which $f_1 \simeq 0 \simeq f_2$.

We hope that this prediction, as well as the rest of our theoretical results, will be checked in the nearest future.

X	π^-	ρ^-	a_1^-
Mass (GeV)	0.139	0.768	1.200
$F_X(GeV)$	0.132	0.318	0.408

Table 4.1: Masses and decay constants of the mesons.

Baryons	Λ_b^0	Ξ_b^0, Ξ_b^-	Ω_b^-	Λ_c^+	Ξ_c^0, Ξ_c^+	Ω_c^0
Mass (GeV) [39]	5.623	5.806	6.059	2.285	2.468	2.699

Table 4.2: Masses of charm and bottom baryons.

Decay	f_1/I	f_2/f_1	α	I	$\Gamma/\,(C_1+\xi C_2)^2\,(GeV)$	$\Gamma\ (GeV)$
$\Lambda_b^0 \to \Lambda_c^+\rho^-$	1.034	0.120	−0.897	0.697	21.30×10^{-15}	26.57×10^{-15}
$\Lambda_b^0 \to \Lambda_c^+ a_1^-$	1.038	0.124	−1.526	0.724	40.85×10^{-15}	50.96×10^{-15}
$\Xi_b^0 \to \Xi_c^+\pi^-$	1.039	0.147	−1.000	0.587	2.759×10^{-15}	3.449×10^{-15}
$\Xi_b^0 \to \Xi_c^+ a_1^-$	1.047	0.154	−1.532	0.639	33.38×10^{-15}	41.65×10^{-15}
$\Xi_b^0 \to \Xi_c^+\rho^-$	1.042	0.149	−0.899	0.605	17.00×10^{-15}	21.27×10^{-15}
$\Xi_b^- \to \Xi_c^0\pi^-$	1.039	0.147	−1.000	0.587	2.759×10^{-15}	3.443×10^{-15}
$\Omega_b^- \to \Omega_c^0\pi^-$	1.049	0.182	0.600	0.497	1.190×10^{-15}	1.485×10^{-15}
$\Omega_b^- \to \Omega_c^0 a_1^-$	1.058	0.191	−0.262	0.554	13.79×10^{-15}	17.21×10^{-15}
$\Omega_b^- \to \Omega_c^0\rho^-$	1.052	0.185	0.561	0.517	7.198×10^{-15}	8.98×10^{-15}

Table 4.3: Nonrelativistic quark model predictions for baryonic form factors, decay widths, and up-down asymmetry α, evaluated at $s = -q^2 = m_X^2$.

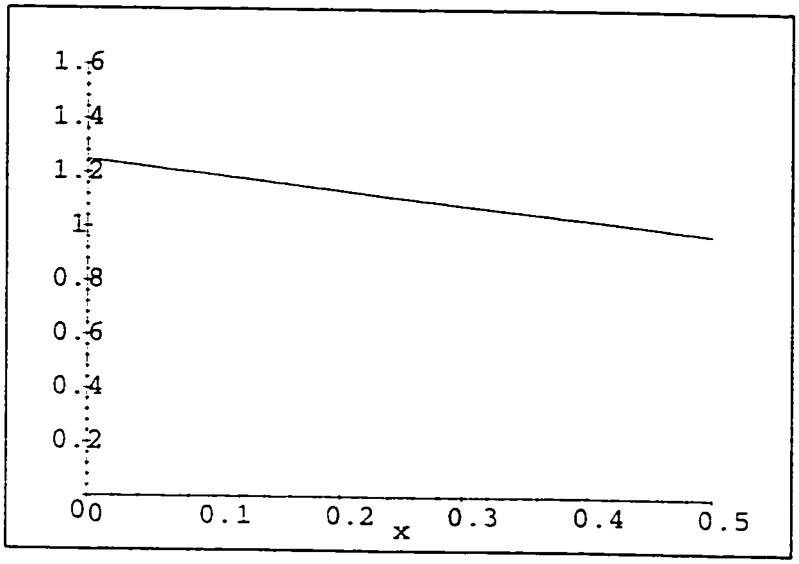

Figure 4.1: Plot of Γ versus $\xi = \text{x}$, where $0 \leq \xi \leq 0.5$.

Chapter 5

Conclusion

In the first part of this study, we have investigated the two-body radiative decays of bottom baryons in the nonrelativistic quark model extending the work done by J. Dey *et al.* on charm baryons [54]. With the obtained decay widths we were able to give an estimate of the following mean lifetimes

$$\begin{aligned}
\tau\left(\Xi_b^{0'}\right) &= 1.36 \times 10^{-20} s \\
\tau\left(\Xi_b^{-'}\right) &= 1.57 \times 10^{-18} s \\
\tau\left(\Omega_b^{*-}\right) &= 1.64 \times 10^{-16} s
\end{aligned}$$

We notice that our prediction for the lifetime of the state Ω_b^{*-}, based on the masses given in [39], is one order of magnitude smaller than that based on the masses given in [41]. The future data would test these predictions.

As for the second part, we have analyzed specific two-body nonleptonic decays of some bottom baryons in the factorization approximation treating the color-matching parameter ξ as a free parameter. In addition, we have used the quark model to determine the weak current baryonic form factors at the desired value $s = m_X^2$, in contrast to the use of the nonrelativistic quark model for the evaluation of form factors at zero recoil $q = 0$ (see [55], [56], and [52]). This zero-recoil approximation does not seem to be justified in the nonrelativistic quark model, since for all the decays considered $|\mathbf{q}| = |\mathbf{p}'| = |\mathbf{p}_c| \simeq 2\ GeV > m_c$ making the c-quark in the daughter baryon B_c relativistic.

The approach adopted by Cheng in [52] also necessitates the exptrapolation of form factors from maximum $s_{\max} = (m_{B_b} - m_{B_c})^2$ to the desired value $s = m_X^2$. In our approach, based on [57], no extrapolation nor any approximation of this kind is needed. The form factors obtained are consistent with the heavy quark spin symmetry and explicitly display $\frac{1}{m}$ correction.

Our predictions for the decay rates and $\alpha-$asymmetries are summarized in Table 4.3. The data is not yet availabe to check these predictions. It is important to note that our predictions for the decay rates are not sensitive to the parameter ξ as shown in Figure 4.1. Thus, these decays are specially suitable to test the other aspects of our method, namely, the quark model which has been used.

The decay rates and $\alpha-$asymmetries are not sensitive to the ratio f_2/f_1 except in one case, namely, the up-down asymmetry of the process $\Omega_b^- \rightarrow \Omega_c^0 a_1^-$. This could constitute a test for the $HQET$ in which $f_2 \simeq 0 \simeq f_1$.

Moreover, we want to stress the fact that we have given the first estimate of the decay width and $\alpha-$asymmetry of nonleptonic weak decay processes involving an axial vector.

Appendix A

Electromagnetic Multipoles

Low lying exited nuclear and particle states usually decay by emitting electromagnetic radiation. This can be described in a series expansion as a superposition of different *multipolarities*. Electric dipole, quadrupole, octupole radiation etc. are denoted by $E1$, $E2$, $E3$, etc. Similarly, the corresponding multipoles are denoted by $M1$, $M2$, $M3$ etc. Conservation of angular momentum and parity determine which multipolarities are possible in a transition. A photon of energy ω and multipoalrity El has an angular momentum l and parity $(-1)^l$, an Ml transition has an angular momentum l and parity $(-1)^{l-1}$.

The lifetime of a state strongly depends upon the multipolarity of the transitions by which it can decay. The lower the multipoalrity, the larger the transition probability. A magnetic transition Ml has approximately the same probability as an electric $E(l+1)$ transition. The decay probability also strongly depends upon the energy. For radiation of multipolarity l, it is proportional to ω_γ^{2l+1} [58].

Appendix B

Dirac Gamma Matrices

B.1 Notation

The notation used in this study is given below (cf. [59] and [35] for example):

$$\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu};\ \mu, \nu = 1, \ldots, 4$$

where

$$\gamma_\mu^\dagger = \gamma_\mu$$

$$\gamma_5 \equiv \gamma_1\gamma_2\gamma_3\gamma_4 = \frac{1}{4!}\varepsilon_{\mu\nu\varrho\sigma}\gamma_\mu\gamma_\nu\gamma_\varrho\gamma_\sigma$$

$$\{\gamma_\mu, \gamma_5\} = 0$$

$$\gamma_5^\dagger = \gamma_5,\ \gamma_5^2 = 1$$

$$a.b \equiv \mathbf{a}.\mathbf{b} + a_4b_4 \equiv \mathbf{a}.\mathbf{b} + (ia_0)(ib_0) = \mathbf{a}.\mathbf{b} - a_0b_0$$

$$\gamma_k \equiv \begin{pmatrix} 0 & -i\sigma_k \\ i\sigma_k & 0 \end{pmatrix};\ k = 1, 2, 3$$

$$\gamma_4 \equiv \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

$$\gamma_5 \equiv \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}$$

where

$$I \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\sigma_{\mu\nu} \equiv \frac{1}{2i}[\gamma_\mu, \gamma_\nu] = -i\gamma_\mu\gamma_\nu$$

$$\sigma_{ij} = \begin{pmatrix} \sigma_k & 0 \\ 0 & \sigma_k \end{pmatrix} ; \; i, j, k = 1, 2, 3 \text{ and cyclic}$$

$$\sigma_{k4} = -\sigma_{4k} = \begin{pmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{pmatrix} ; \; i, j, k = 1, 2, 3 \text{ and cyclic}$$

B.2 Useful Relations

$$Tr\,(\Gamma) = 0$$

where

$$\Gamma \equiv \gamma_\mu, \sigma_{\mu\nu}, i\gamma_5\gamma_\mu, \gamma_5.$$

$$(\gamma.a)(\gamma.b) = 2a.b - (\gamma.b)(\gamma.a)$$

$$Tr\,(\gamma_\mu\gamma_\nu) = Tr\,(\gamma_\nu\gamma_\mu) = 4\delta_{\mu\nu}.$$

$$Tr\,(\gamma_\mu\gamma_\nu\gamma_\varrho\gamma_\sigma) = 4(\delta_{\varrho\sigma}\delta_{\mu\nu} - \delta_{\nu\sigma}\delta_{\mu\varrho} + \delta_{\mu\sigma}\delta_{\nu\varrho})$$

For an odd number of γ matrices

$$Tr\,(\gamma_\mu\gamma_\nu...\gamma_\varrho) = 0.$$

For less than 4 γ matrices

$$Tr\left(\gamma_5\gamma_\mu\cdots\right) = 0,$$

while for 4γ

$$Tr\left(\gamma_5\gamma_\mu\gamma_\nu\gamma_\varrho\gamma_\sigma\right) = 4\varepsilon_{\mu\nu\varrho\sigma}.$$

$$Tr[(\gamma.a)(\gamma.b)] = 4a.b$$

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